Project 1.10

Verification of Infrastructure Networks

Scalable Verification Methodologies for Complex Infrastructure Networks

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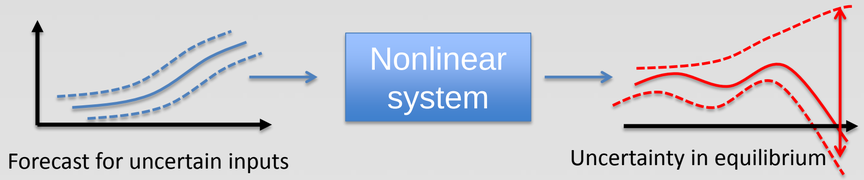
Objective

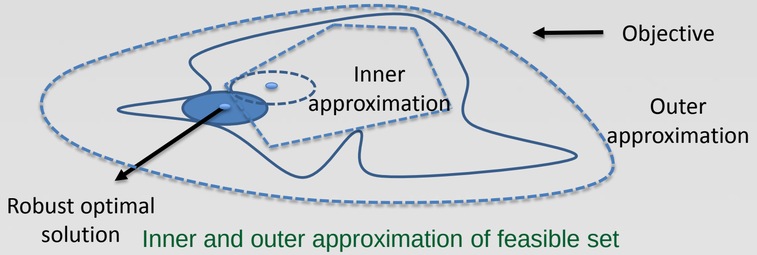
- Verify existence and robustness of desirable equilibria in nonlinear infrastructure networks with uncertain inputs
- Develop algorithms for robust optimization and control of infrastructure networks
- Study computational complexity and conservatism of developed approaches



Fragility of nonlinear systems under uncertainty

Methodology

- Use computational topology theory and convex optimization to develop tractable sufficient conditions for existence of robust equilibria
- Test and refine the algorithms on uncertain power system test cases



Inner and outer approximation of feasible set

Solvability certificate

In steady state, infrastructure networks can be modeled as parameterized quadratic equations

$$F(x,u) \equiv Q(x,u) + L(x,u) + c(u) = 0$$

Quadratic in x, affine in u

Theorem. *Let* $J(x;u) = \frac{\partial F}{\partial x}, J_{\star\star} = J(x^\star;u^\star)$. *Suppose* $J_{\star\star}$ *is invertible. Define:*

$$e(u) = \left\| (J_{\star\star})^{-1} F(x^\star, u) \right\|$$

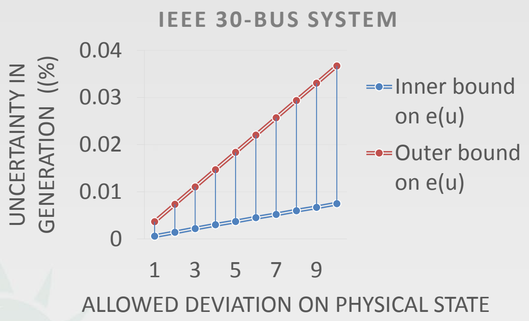
$$h_\star = \max_{\|y\| \le 1} \left\| (J_{\star\star})^{-1} Q(y) \right\|$$

Then for any $r > 0$, $F = 0$ *has a solution in the set* $\{x : \|x - x^\star\| \le r\}$ *if*

$$e(u) \le \max\left(r(1 - r h_\star), \min\left(\frac{1}{4h_\star}, r^2 h^\star \right) \right)$$

Theorem that constructs inner approximation of feasible set
Can be used to verify existence of robust equilibrium

Numerical evaluation



Inner and outer bounds on the robustness margin of a 30 bus power system as a function of allowed deviation on physical state

Algorithm can provide tight estimates of robustness margin for a given power system equilibrium

Exact solvability certificate

Theorem. *For power distribution networks:*
(1) A unique stable equlibrium exists or no solutions exist
(2) The set of parameters for which a solution exists is convex
(3) Robust feasibility wrt uncertain parameters can be easily checked

Robust feasible set can be characterized precisely for distribution networks

Future work

- Robust optimization algorithms for infrastructure networks
- Analyze interdependent infrastructure networks (gas-grid, transport-grid) with compositional certificates
- Scalable open source implementation

Publications:

1. High-voltage Solution in Radial Power Networks: Existence, Properties and Equivalent Algorithms. Krishnamurthy Dvijotham, John Simpson-Porco, Enrique Mallada. Accepted to appear in *IEEE Control Systems Letters*
2. Solvability Regions of Affinely Parameterized Quadratic Equations. Krishnamurthy Dvijotham, Hung Nguyen, Konstantin Turitsyn. Accepted to appear in *IEEE Control Systems Letters*
3. A Framework for Robust Steady-State Voltage Stability of Distribution Systems. Krishnamurthy Dvijotham, Hung Nguyen, Konstantin Turitsyn. Under review at *IEEE Transactions on Smart Grid*

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