Project 1.7

Quantifying Controllability and Observability in Nonlinear Infrastructure Networks

An Input-Output Framework for Engineering Distributed Control Architectures

Enoch Yeung, Soumya Kundu, Panos Stinis, Ben Romano, Craig Bakker, Nathan Hodas

CCSI Control of Complex Systems Initiative

Pacific Northwest NATIONAL LABORATORY

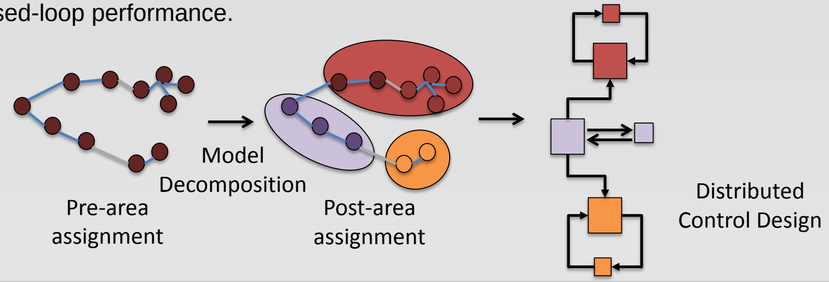
Proudly Operated by Battelle Since 1965

U.S. DEPARTMENT OF ENERGY

PNNL-SA-XXXXX

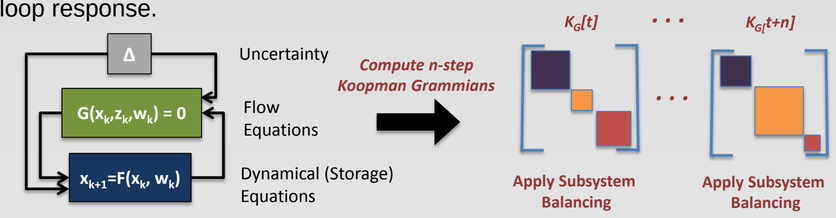
Objective

- Develop model decomposition methods that enable design and execution of distributed control architectures on infrastructure systems with *nonlinearity, uncertainty,* and *switching.*
- Enable characterization of critical interdependencies between infrastructure systems.
- Evaluate how model decomposition methods impact system dynamics, control, and closed-loop performance.



Methodology

- Extend Koopman operator theory for input-actuated *nonlinear* discrete-time and continuous-time dynamical systems.
- Learn an input-Koopman operator from experimental data or an *a priori* model.
- Utilize Koopman operators to compute generalized Koopman-grammians.
- Perform subsystem balancing and generate BIBO stability certificates for closed-loop response.



Grammian decomposition enables discovery of BIBO-stable distributed control structure

Koopman Grammians Are Consistent with Nonlinear Controllability and Observability

When modeling transient dynamics, infrastructure networks can be modeled with the Koopman equations: $\psi_x(x_{n+1}) = \mathcal{K}_x\psi_x(x_n) + \mathcal{K}_u\psi_u(x_n, u_n)$

We define the Koopman controllability and observability grammians as:

$$X_c^\psi = \sum_{j=0}^{n-1} \Phi_c^\psi (\Phi_c^\psi)^T = \sum_{j=0}^{n-1} \mathcal{K}_x^j \mathcal{K}_u \mathcal{K}_u^T (\mathcal{K}_x^j)^T \qquad X_o^\psi = \sum_n (\Phi_\psi^y)^T \Phi_\psi^y = \sum_n (\mathcal{K}^n)^T W_h^T W_h \mathcal{K}^n$$

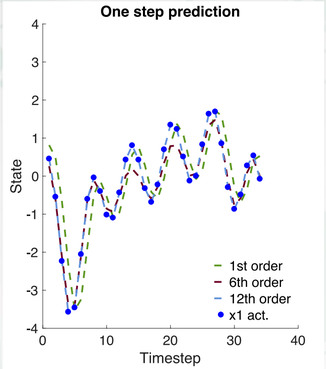
Lemma 2: Suppose the discrete time nonlinear system

$$x_{k+1} = F(x_k, u_k) = f(x_k) + g(u_k) \qquad (47)$$

is locally controllable, i.e. it generates an involutive distribution D^* of rank n. Then the system has a positive definite Koopman controllability grammian.

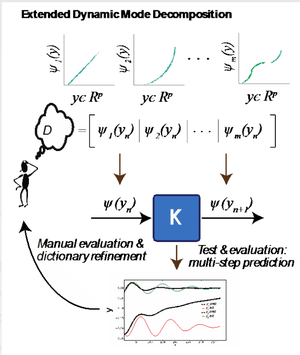
Koopman Balanced Truncation Enables Model Reduction

The Koopman grammians can be used to perform balanced truncation to obtain a reduced model that evolves linearly over the Koopman observable space.

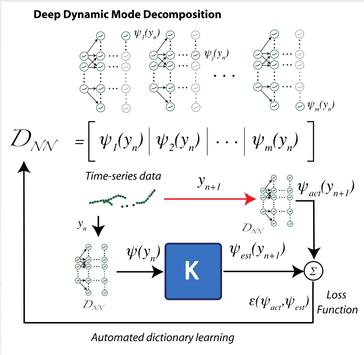


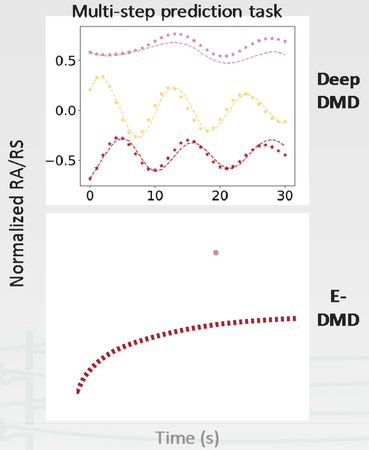
Deep Koopman Operators Improve Multi-step Prediction on the IEEE 30 Bus Benchmark System

Current Approach



Our Novel Approach





Future Work & References:

- Developing algorithms for computing stochastic & hybrid Koopman operators for infrastructure systems with uncertainty and/or switching dynamics.
- Investigate stochastic singular perturbation and novel time-scale separation approaches towards model reduction.

[1] A Koopman Operator Approach to Computing and Balancing Grammians for Discrete Time Nonlinear Systems, submitted to the 2017 IEEE Conference on Decision and Control
[2] Learning Deep Neural Network Representations for Koopman Operators of Nonlinear Dynamical Systems, manuscript in preparation.