CCSI Control of Complex Systems Initiative

Pacific Northwest NATIONAL LABORATORY

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Control of Complex Systems:
An Integrated Perspective on Modern Power Grid Control

Assessing Flexibility in Uncertain Frequency Responsive Loads

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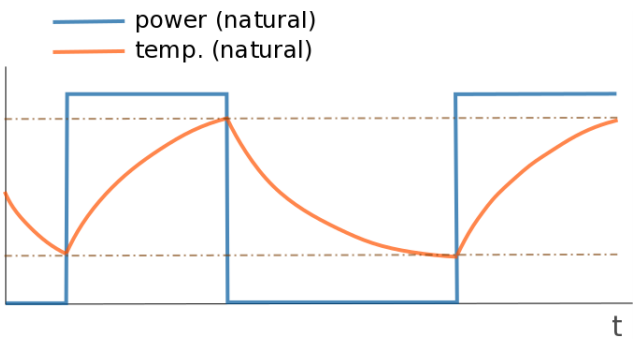
Frequency Responsive Loads: Basics

- Consider switching loads toggling between *on* and *off* states
  - "*energy*-loads": quality of service depends on energy consumed over a time
  - Flexibility in changing their power consumption instantly (for a short duration)

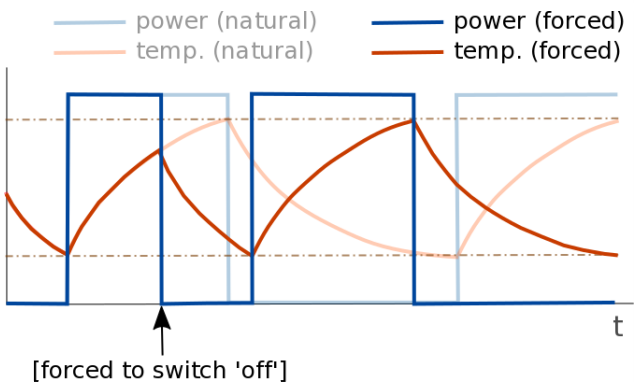
$$(\text{internal dynamics}) : \dot{\theta}(t) = -a(t)\theta(t) + b(t) + s(t)\eta P\,,$$

$$(\text{switching logic}) : s(t^+) = \begin{cases} 0\,, & h(\theta(t)) \geq \delta/2 \\ 1\,, & h(\theta(t)) \leq -\delta/2 \\ s(t)\,, & \text{otherwise} \end{cases}$$

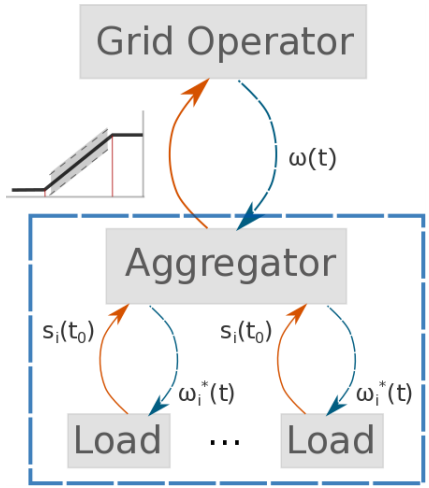


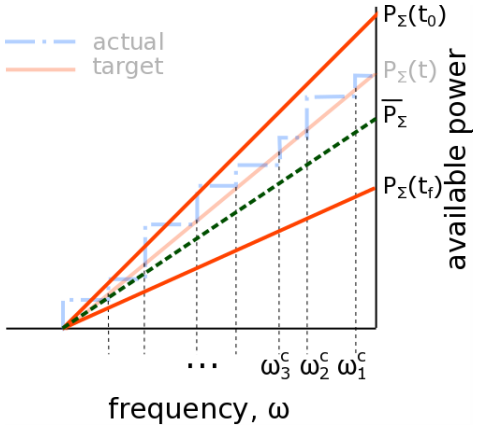
- A load can sign up for frequency response
  - (Possibly) through a load aggregator
  - Gets assigned a frequency threshold

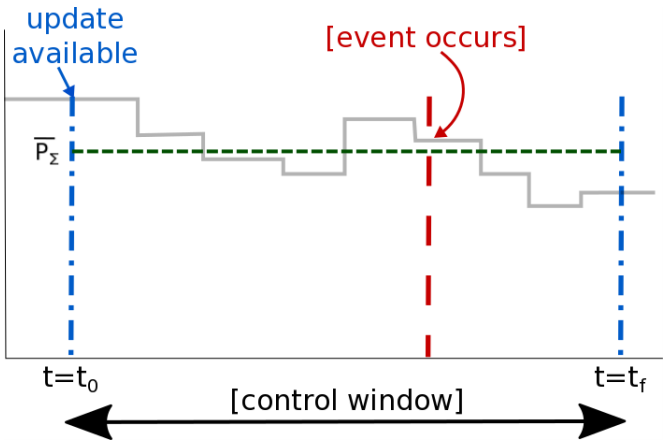
$$(\text{under-frequency}) : s(t^+) = \begin{cases} 0\,, & \omega(t) < \omega^c(t) \ \&\ s(t) = 1 \\ s(t)\,, & \text{otherwise} \end{cases}$$



Coordinated Ensemble Response







$$P_\Sigma(t) = \sum_i s_i(t) P_i \qquad \xi(t|t_0) := \frac{\left|P_\Sigma(t) - \overline{P_\Sigma}\right|}{\overline{P_\Sigma}}$$

- Aggregator assigns frequency thresholds to provide a *droop* response
- The availability of the responsive loads varies over the control window
  - Updates on the device states are obtained (only) at the start
  - Local constraints (e.g., temperature limits) take precedence
- What is the *optimal demand flexibility* that minimizes response error?

Optimal Demand Flexibility

- Expected squared response error: $\mathbb{E}\{\xi(t|t_0)^2\}, \quad \xi(t|t_0) := \frac{\left|P_\Sigma(t) - \overline{P_\Sigma}\right|}{\overline{P_\Sigma}}$
- Define the optimal flexibility as:

$$\overline{P_\Sigma}^* := \arg\min_{\overline{P_\Sigma}} \left[\sup_{t\in\mathcal{C}} \mathbb{E}\left\{\xi(t|t_0)^2\right\}\right]$$

  - i.e., minimizes the maximal expected error over the control window
- Observe that
  - For small control window, probability of availability is monotonic: $p_{on}(t) \;\; \forall t \in \mathcal{C}$
  - Maximal expected error occurs at either end of the control window:

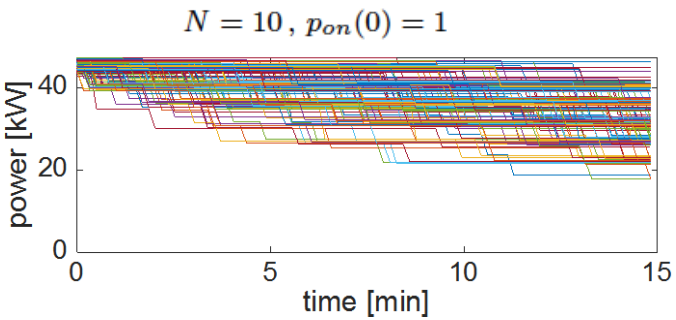
$$\forall t \in \mathcal{C} = [t_0, t_f): \quad \mathbb{E}\{\xi(t|t_0)^2\} \leq \sup_{t=\{t_0, t_f^-\}} \mathbb{E}\{\xi(t|t_0)^2\}$$

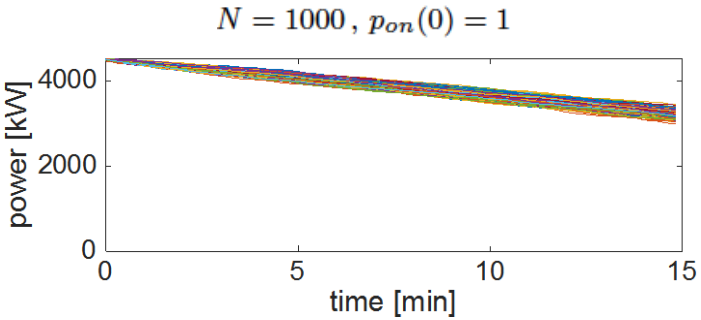
- Theorem: The optimal flexibility is given by

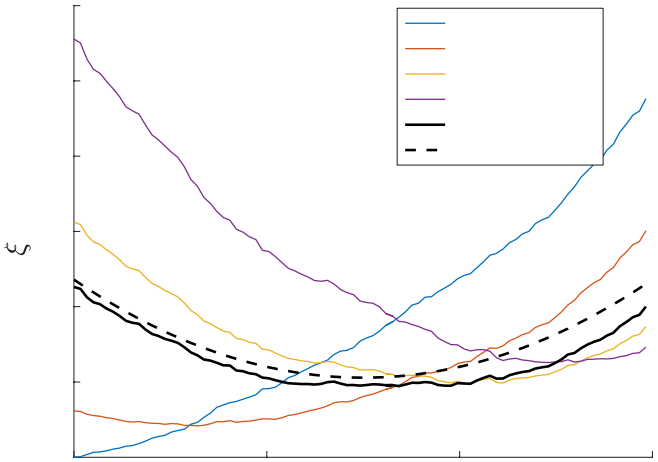
$$\overline{P_\Sigma}^* = \frac{\langle P^2 \rangle}{2\langle P \rangle} + (N-1)\frac{(p_{on}(t_0) + p_{on}(t_f))}{2}\langle P \rangle.$$

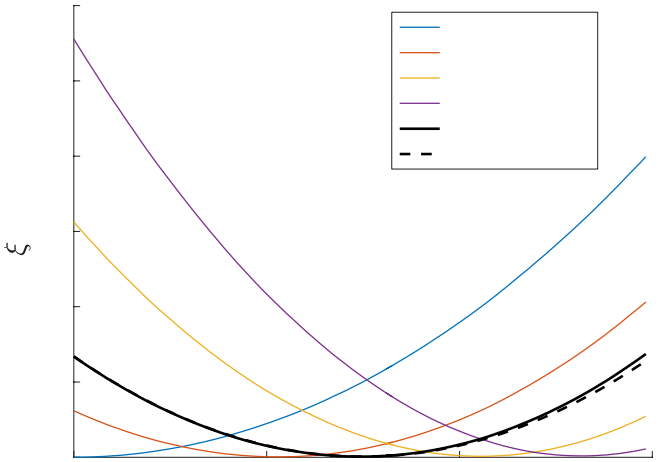
$$\text{where} \quad \langle P \rangle := \mathbb{E}\left[P_i\right] \; \forall i \quad \text{and} \quad \langle P^2 \rangle := \mathbb{E}\left[P_i^2\right] \; \forall i$$

Simulation: Electric Water Heaters









Concluding Remarks

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- Hierarchical framework for autonomous frequency response
  - An aggregator coordinates the net flexibility in an ensemble of loads
  - Each load is assigned a (unique) frequency threshold for response
  - Local end-user constraints (e.g., temperature limits) take precedence
- We provide a closed-form expression for the optimal assessment of flexibility in the ensemble of loads, that minimizes response error
  - under uncertainties in load dynamics
- Future work will address
  - Integration issues of such control schemes in the grid operation
  - Value in relaxing of local constraints (with penalty)
  - Effect of control action on future availability (multi-period)

Thank you!

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