



# ***Tutorial: Modeling Frameworks to Enable New Opportunities in Grid Optimization and Control***

*Professor Chris DeMarco*

*Department of Electrical & Computer Engineering*

*University of Wisconsin-Madison*

*Control of Complex Systems: An Integrated*

*Perspective on Modern Power Grid Control*

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# Acknowledgement

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- *Today's tutorial includes a Sparse Tableau formulation of the Optimal Power Flow, based on work in collaboration with University of Wisconsin-Madison colleague Professor **Michael Ferris**, and PhD candidates **Byungkwon Park** and **Jayanth Netha**.*
- *This formulation is being employed in the construction of large-scale synthetic grid models for OPF, as part of the EPIGRIDS project under the ARPA-E GRID DATA program. This work is supported by the Advanced Research Projects Agency-Energy (ARPA-E), U.S. Department of Energy, under Award Number DEAR0000717.*
- *This U.S. Department of Energy support is gratefully acknowledged; however, views and opinions of the author expressed in the following presentation do not necessarily state or reflect those of the United States Government or any agency thereof.*



# ***Motivation for Today's Tutorial***

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- *Electric power systems optimization and control have long, well-studied history. Many excellent texts and tutorial, carefully developing grid models from underlying first principles.*
- *Natural tendency for researchers focusing on optimization and control to accept well-established choices in component modeling, network representation, and coordinates.*
- *However, experience in many other areas suggest that advanced optimization and control applications often demand close interplay with domain-specific modeling.*



# ***Motivation for Today's Tutorial***

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- *Important to recognize that “textbook” choices of power system representations inevitably made in context of specific component technologies, operational objectives, and computational tools.*

*Obvious observation follows, motivating today's talk:*

- *As the grid sees rapid changes in (at least some of... ) its component technologies, operational objectives, and computational tools, approaches to grid modeling benefit from re-examination.*



## ***Disclaimer for Today's Tutorial***

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- *Little here that does not follow directly from concepts that could be found Desoer & Kuh's 1969 text, **Basic Circuit Theory**. No challenges to underlying first principles.*
- *Rather, an examination of other “forks along the road,” in moving from those first principles, toward the most effective models and representations.*
- *Accepting George Box's maxim, we'll (mostly) step over question of “my model, right or wrong,” and straightway consider usefulness among families of grid models.*



# ***Synopsis for Today's Tutorial***

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*Three related topics, each with a “revisionist” twist*

- *Component modeling. Approach here will stress use multi-ports as underlying ideal circuit elements in grid model, maintaining element port voltages and currents.*
- *Network representation. Abandon “Ybus.” In power systems parlance, we’ll ditch the bus-branch model. Seek a streamlined approach to node-breaker models. Advocate Sparse Tableau Analysis (STA) for network constraints consistent with node-breaker detail.*



# ***Synopsis for Today's Tutorial***

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*Third topic:*

- *Coordinate Choice, in OPF: With complex phasor bus voltages and powers, one obviously has choice of polar or rectangular coordinates. STA approach extends this coordinate frame choice to a more complete set of bus **and component** currents, voltages, and powers.*
- *Coordinate Choice, in Dynamic Analysis for Control: Power models have long employed representations in which the states associated with a bus voltage are the magnitude and phase angle of its fundamental-frequency Fourier coefficient. High penetration power electronics suggests value in multi-frequency generalizations.*



## ***Topic 1: Multi-ports***

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- *Claim: texts often “handicap” the power flow model development in choice of admissible ideal elements. Case in point: Overhead three-phase transmission lines (below: 69kV line at Madison, WI Blount St power plant)*





# ***Topic 1: Two-Ports and Multi-ports***

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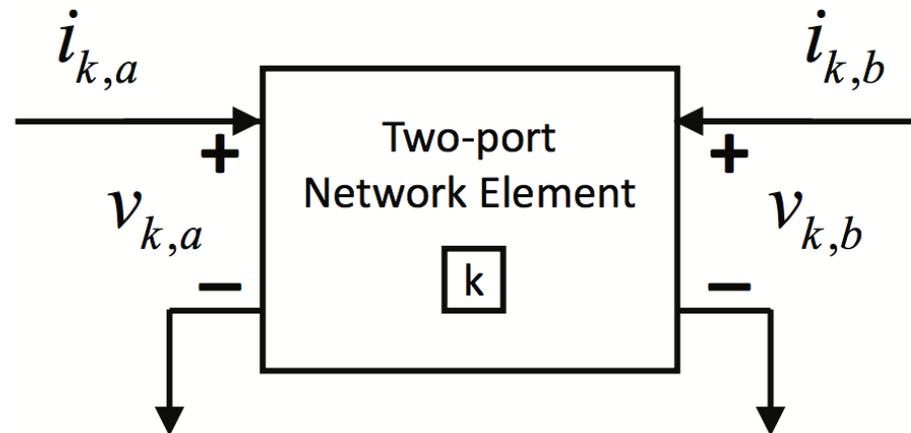
- *Transmission lines are perhaps the most ubiquitous power system component, and the very definition of distributed-parameter.*
- *Consider typical textbook's first analysis steps:*
  - (i) *Begin from pde's describing distributed behavior.*
  - (ii) *Impose assumptions of balanced three phase operation, in sinusoidal steady state (SSS).*
  - (iii) *Focus on relation between "sending end" and "receiving end" voltage-current pairs.*

*(BTW – these first steps are perfectly ok, when assumptions hold appropriately)*



# Transmission line as a two-port

- Assumption (ii) provides per-phase algebraic relations; (iii) dictates structure of relation is **naturally** a two-port.

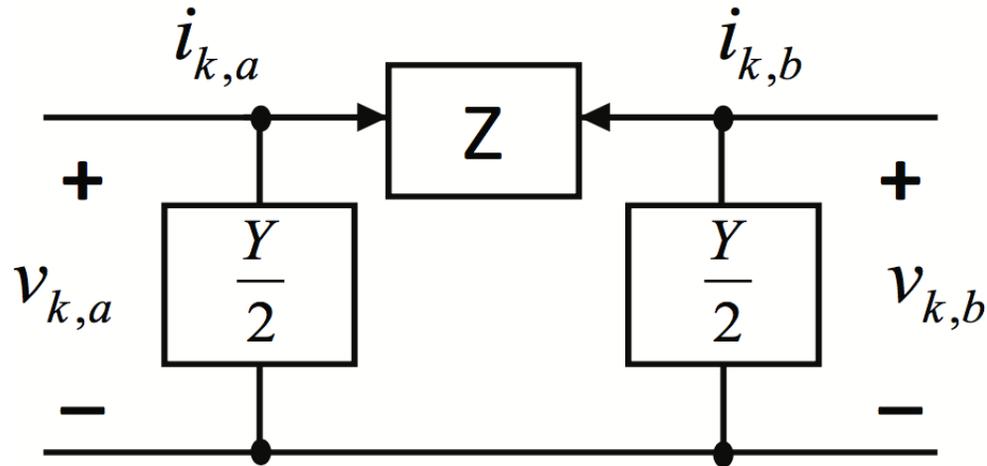


- The handicap (IMO) standard power systems formulation occurs in next step: creation of an equivalent circuit for this two-port, constructed of **strictly two-terminal admittances** (instead of keeping the two-port model)



## ***Shortcoming of Pi-equivalent for Transmission Line Two-port***

- *Characteristics of overhead line yield  $Y$  and  $Z$  below, that match behavior of distributed model in SSS, at terminals.*



- *Standard practice line specifies data for OPF via  $Y$  and  $Z$ . Shortcoming: Otherwise “reasonable-looking” ( $Y,Z$ ) can fail to be realizable from true, physical parameters of conductor radius, permeability, and inter-phase conductor distance.*



## ***Ideal Transformer as a Two-Port***

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- *An ideal transformer the poster-child for two-port analysis. For transformer having transformation gain “k,” the two constitutive relations among the port variables are simply*

$$v_b = kv_a, i_b = (1/k^*)i_a$$

*(for phase shifting transformer, k may be complex)*

- *But standard practice in specifying power flow/OPF data disallows a “pure,” ideal transformer as a power systems element. Non-ideal series reactance **must** be included to represent the physical effect of leakage flux.*
- *Why?*



# ***Ideal Transformer as a Two-Port***

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- *Here the problem involves both the nature of the individual component's analysis, and the formulation of the overall network constraints.*
- *While details will follow, the insistence on Ybus analysis (strict nodal analysis) requires that constitutive relations for every component must have the property that the component's current(s) be expressible in terms of the component's voltage(s). From a two-port perspective, the component must permit an admittance representation.*
- *An ideal transformer does not have this property.*



# General Two-Port Element Representation

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- *General representation of a two-port is straightforward. In the a nonlinear case, assuming phasor quantities, a two-port element imposes two complex constraints on the four complex port variables, i.e.*

$$f_k : \mathbb{C}^4 \longrightarrow \mathbb{C}^2$$
$$f_k(v_{k,a}, i_{k,a}, v_{k,b}, i_{k,b}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (1)$$

- *For affine-linear case, most prevalent in power systems, the general two-port written as: (for strictly linear,  $u_s=0$ )*

$$F_v v + F_i i = u_s \quad (2)$$

- *Ybus-based OPF formulations restrict to linear elements, and require that  $F_i$  must be invertible.*



# ***The Role of Circuit Breakers***

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- *Circuit breakers (i.e., switches) are also ubiquitous throughout the power grid. In recent years, considerable attention in OPF literature on treatment of line switching problem.*
- *In line switching problem, position of circuit breakers on select transmission lines are allowed to be integer decision variables*
- *Easily accommodated in Ybus/admittance formulation: one simply sets admittances of a line's pi-equivalent to zero when that line's breaker is open.*
- *But circuit breakers have other important roles.*



# ***The Role of Circuit Breakers***

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- *Circuit breakers also “sectionalize” buses in a substation.*

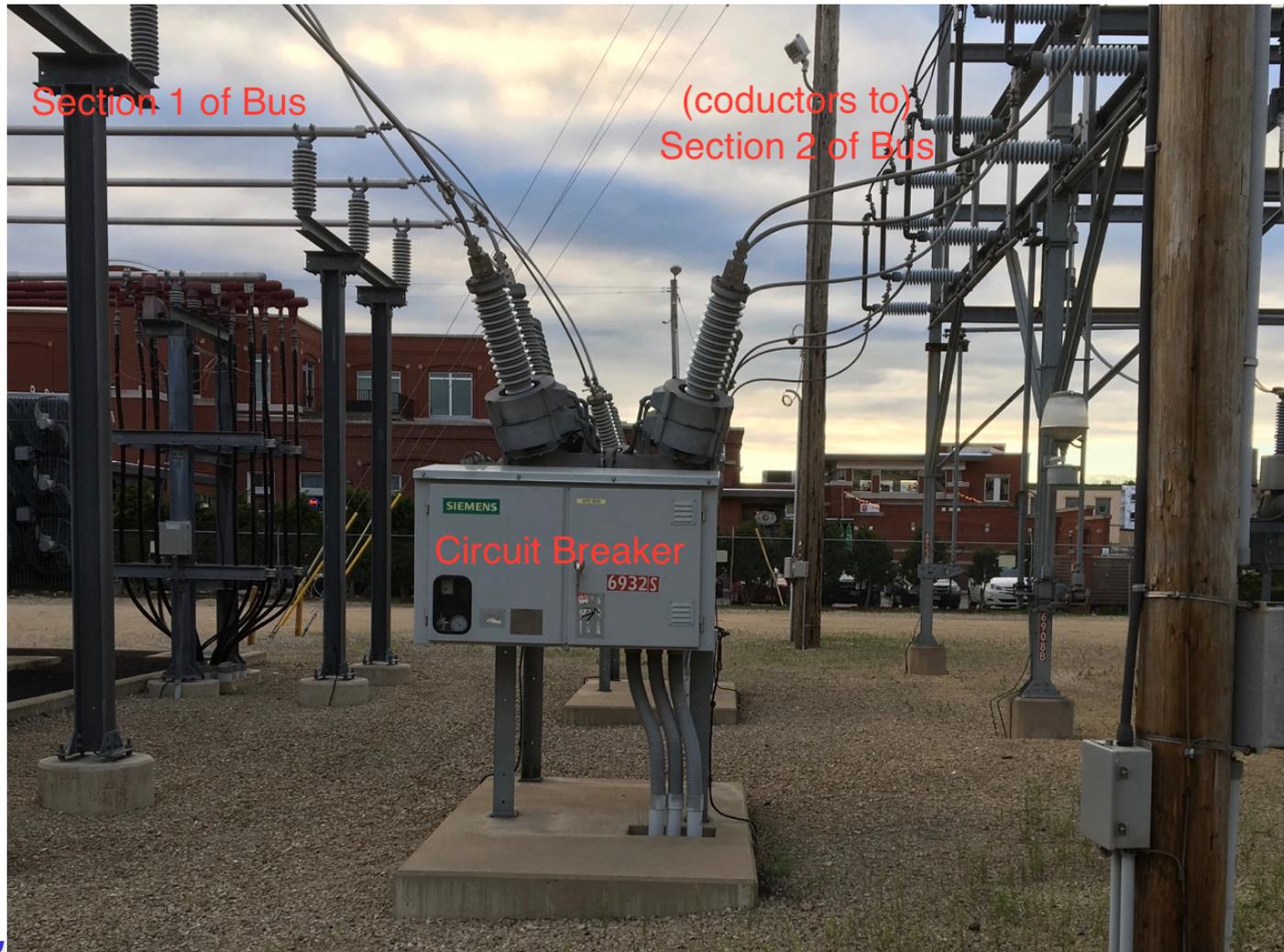




# ***The Role of Circuit Breakers***

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- *Circuit breakers also “sectionalize” buses in a substation.*





# ***The Role of Circuit Breakers***

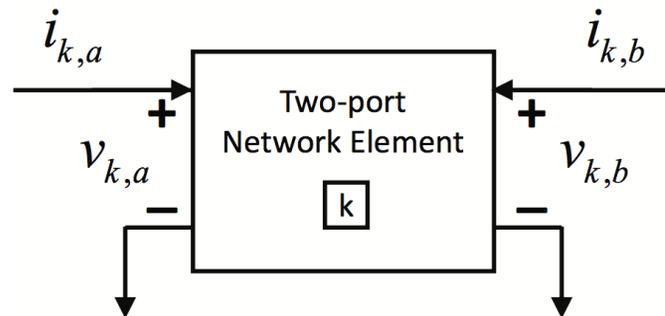
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- *Breaker Closed: two sections of bus held to equal voltage, function as a single node in the idealized circuit.*
- *Breaker Open: zero current flows through breaker, two sections of bus function as two independent nodes.*
- *Standard power flow/OPF models, based on strict nodal analyses, use **ONLY** node voltages as fundamental circuit variables. Hence, they change dimension of model between the two breaker positions.*
- *In power systems parlance, a “topology processing” algorithm rebuilds a distinct  $Y_{bus}$  admittance matrix for each configuration. Editorial comment: IMO, this is dumb.*



# Circuit Breaker as ANOTHER Natural Two-Port

- (i) breaker position indicated by binary variable  $\gamma$ ;
- (ii) maintain port voltage/current pairs as explicit variables;
- (iii) as previously described, don't insist on  $F_i$  invertible.



$$\text{Circuit breaker closed, } \gamma = 1 : \begin{cases} v_a - v_b = 0 \\ i_a + i_b = 0 \end{cases}$$

$$\Rightarrow \underbrace{\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}}_{\mathbf{F}_v} \begin{bmatrix} v_a \\ v_b \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}}_{\mathbf{F}_i} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



# ***Circuit Breaker as ANOTHER Natural Two-Port***

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Circuit breaker open,  $\gamma = 0$  :  $\begin{cases} i_a = 0 \\ i_b = 0 \end{cases}$

$$\Rightarrow \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\mathbf{F}_v} \begin{bmatrix} v_a \\ v_b \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{F}_i} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

*and as single description, in terms of  $\gamma$ :*

$$\underbrace{\begin{bmatrix} \gamma & -\gamma \\ 0 & 0 \end{bmatrix}}_{\mathbf{F}_v} \begin{bmatrix} v_a \\ v_b \end{bmatrix} + \underbrace{\begin{bmatrix} (1-\gamma) & 0 \\ \gamma & 1 \end{bmatrix}}_{\mathbf{F}_i} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

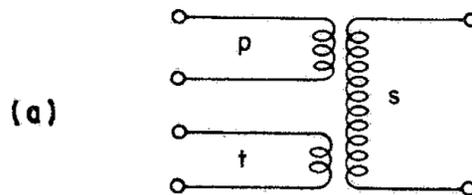


# And Three-winding Transformers as Natural Three-Ports

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- *Classic textbook presentation (B.M. Weedy, Electric Power Systems):*

*Three-winding transformers* Many transformers used in power systems have three windings per phase, the third winding being known as the tertiary. This type can be represented under balanced three-phase conditions by a single-phase equivalent circuit of three impedances star-connected as shown in Figure 3.40. The values of the equivalent impedances  $Z_p$ ,  $Z_s$  and  $Z_t$  may be obtained by test. It is assumed that the no-load currents are negligible.





# And Three-winding Transformers as Natural Three-Ports

- From a circuit analysis viewpoint, this **cries out** for representation as a multi-port. But instead...

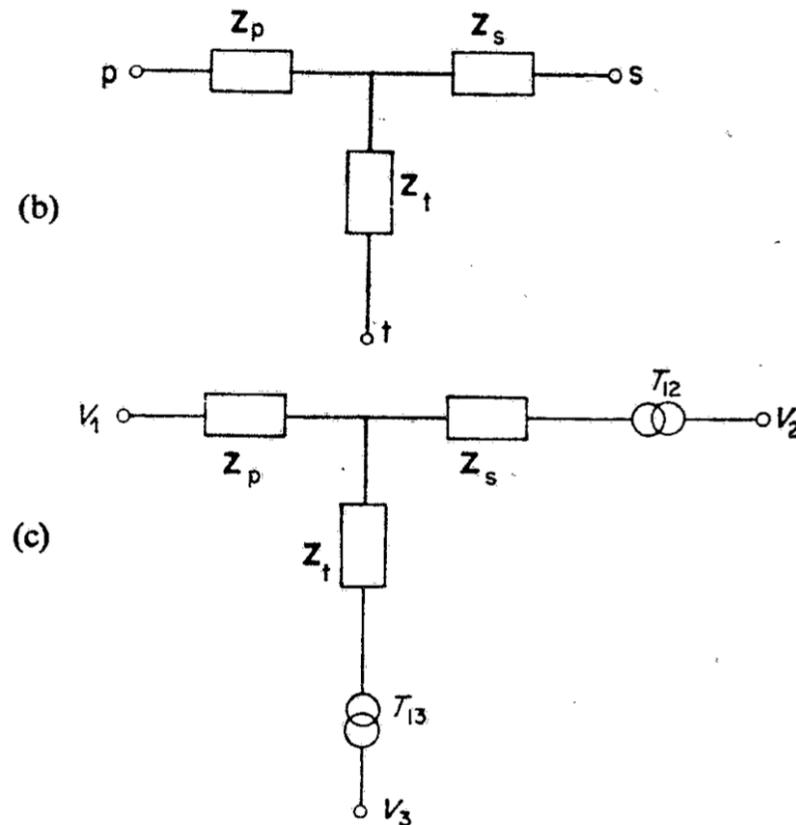


Figure 3.40(b) and (c) Equivalent circuits.



## ***And Three-winding Transformers as Natural Three-Ports***

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- *Even Weedy basically admits this is a dumb idea. In particular:*

It should be noted that the star point in Figure 3.40b is purely fictitious and that the diagram is a single-phase equivalent circuit. In most large transformers the value of  $Z_s$  is very small and can be negative. All impedances must be referred to common volt-ampere and voltage bases. The complete equivalent circuit is shown in Figure 3.40(c).

- *Instead, three-port description:*

$$\underbrace{\begin{bmatrix} \frac{1}{N_a} & -\frac{1}{N_b} & 0 \\ 0 & \frac{1}{N_b} & -\frac{1}{N_c} \\ 0 & 0 & 0 \end{bmatrix}}_{\mathbf{F}_v} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ N_a & N_b & N_c \end{bmatrix}}_{\mathbf{F}_i} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



# ***Network Interconnection Constraints: KCL and KVL***

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- *At risk of rehashing a sophomore circuit course, observe that thus far we have described only the constitutive relations for a set of ideal elements. **These pertain to the elements themselves, independent of interconnection topology.***
- *When elements are interconnected in a network, linear KCL and KVL constrain those elements' port currents and voltages, and relate them to nodal quantities:*
  - (i) *node voltages,  $V$  (in the grid, busbar voltages);*
  - (ii) *currents externally injected at nodes,  $I$ ;*  
*(these represent currents injected by generation, or withdrawn by load; descriptions of  $I$  behavior to follow)*



# ***Network Interconnection Constraints: KCL and KVL***

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- *Familiar mechanism to express KCL and KVL in compact form is that of node-to-branch incidence matrix, here denoted,  $A$ .*
- *Combining KCL, KVL, and linear constitutive relations, Sparse Tableau formulation is extraordinarily simple:*

$$Ai = I$$

$$v = A^T V$$

$$F_v v + F_i i = 0$$

- *If generation and load behaved as constant current sources/sinks, with fixed  $I$ , we'd be done now.*



# ***Network Interconnection Constraints: KCL and KVL***

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- *The strict nodal analysis of  $Y_{bus}$  is easily recovered as a special case reduction of the Sparse Tableau.*
- *One simply eliminates the “intermediate variables” of elements’ port currents and voltages,  $i$  and  $v$ , to obtain the relation between externally injected currents and bus voltages as:*

$$I = \underbrace{-A \cdot (F_i)^{-1} \cdot F_v \cdot A^T}_{Y_{bus}} V$$



# ***Generation and Load Behavior***

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- *Behavior of transmission network is well-modeled in linear relations between currents and voltages. (even if we introduce nonlinearities via variable topology, through breaker position  $\gamma$ ).*
- *Nonlinearities and non-convexity in current vs. voltage relations appear when modeling behavior of generation and load.*
- *One may treat generation and load as general nonlinear one-ports, connecting each node/bus to ground. Here we choose to restrict to a special case, treating them as nonlinear current sources, setting  $I$ .*



# ***Generation and Load Behavior***

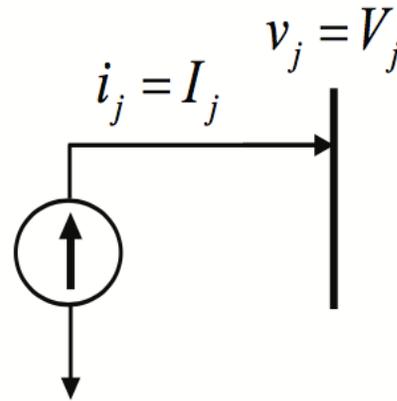
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- *Confession: exactly the same type of limitation just criticized in Ybus formulation: restrict each generator or load to be a voltage-controlled element.*
- *Rationalization: physical equipment that dominates power production and consumption today is largely inductive in nature (e.g., coupled windings of a synchronous generator or induction motor). Voltage as input, current as state and/or output is therefore natural.*
- *Caveat: As voltage source inverters play growing role interfacing future generation to grid, we'll want to discard this voltage-controlled current source restriction.*



# Generation and Load Behavior

- In OPF, most common nonlinear behavior associated with generation or loads is that of constant complex powers, denoted  $S$ , either as fixed parameters, or as decision variables to be solved via optimization.*



Here  $I_j - \frac{S_j^*}{V_j^*} = 0$  or power balance form,  $S_j = V_j I_j^*$



# ***Generation and Load Behavior***

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- *In setting for control design, generators (and increasingly, loads) are most often the dynamic elements at which control is exercised.*
- *In this context, current  $I$  will be the output quantity associated with the generator or load, bus voltage  $V$  will serve as an input (perhaps along with other externally applied inputs).*
- *Synchronous generators provide common example of this structure, as described on following slide.*



# ***Generation and Load Behavior***

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- *External inputs,  $u$ , typically comprised of exciter setpoint, and applied mechanical torque on generator shaft. States,  $x$ , typically currents or fluxes of windings, along with mechanical shaft angle and angular velocity.*
- *Then, at each bus  $k$ , local set of state & output equations, coupling to network through  $V_k$  and  $I_k$*

$$\dot{x} = f(x, u, V_k)$$

$$I_k = g(x, u, V_k)$$

- *This same general structure applies for variety of dynamic generation or load elements, assuming “bus voltage as input, current as output” appropriate.*



# ***Real-Valued Coordinate Choices for Phasor Based Models***

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- *Phasor-based analysis offers inherent flexibility in choice of real-value coordinate representation of complex quantities. Most naturally: polar versus rectangular.*
- *Ybus/Nodal analysis formulations use only bus voltages as key network electrical variables. Hence pros/cons of polar versus rectangular coordinate choice has tended to focus exclusively on bus voltages.*
- *However, because it maintains “intermediate” port currents and voltages as additional phasor quantities its formulation, Sparse Tableau offers this choice over larger set of variables.*



# ***Real-Valued Coordinate Choices for Phasor Based Models***

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- *Choice of coordinate system can have significant impact on geometry of the feasible region for the OPF, sometimes significantly impacting performance of optimization algorithms.*
- *Editorial comment: Better understanding of geometry of feasible region for the OPF, and the interplay between this geometry and modeling/coordinate system choices, is (IMO) a critical area for research.*



# Experience with Sparse Tableau Formulation OPF

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*Sparse Tableau offers very simple (dare I say elegant?)  
formulation of OPF, as summarized below:*

$$\min_{P, Q, v, i, V, I} \sum_{j \in \mathbf{G}} \tilde{c}_j (P_{g,j}) \quad \text{subject to}$$

Linear Element:  $F_v v + F_i i = 0$

KCL:  $I - Ai = 0$

KVL:  $v - A^T V = 0$

Nonlinear Element:  $S - V \odot (I)^* = 0$

Gen. Limit:  $P_j^{min} \leq P_{g,j} \leq P_j^{max}$

$$Q_j^{min} \leq Q_{g,j} \leq Q_j^{max}, \forall j \in \mathbf{G}$$

Vol. Limit:  $V_j^{min} \leq |V_j| \leq V_j^{max}, \forall j \in \mathbf{N}$

Line Limit:  $|i_{k,a/b}|^2 \leq i_k^{max}, \forall k \in \mathbf{L}$



# ***Computational Experience with Sparse Tableau Formulation OPF***

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- *Historically, circuit software tools have preferred Modified Nodal Analysis. MNA maintains more variables than strict nodal analysis, but fewer than Sparse Tableau. So why advocate Sparse Tableau in OPF?*
- *Port currents are constrained quantities. If not kept in the variable set, they must be computed anyway.*
- *With advances in NLP solvers, algorithms becoming good at efficiently choosing intermediate variables to eliminate in their solution process. Suggests we might do better not a priori **forcing** choice of which variables to eliminate, but rather let the solver make these choices.*



# ***Computational Experience with Sparse Tableau Formulation OPF***

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- *Experiments comparing Sparse Tableau to traditional Ybus OPF formulations are very preliminary, and to date have been performed only in the GAMS general purpose optimization environment.*
- *In a half dozen test systems from the MATPOWER distribution, up to several thousand buses, experience so far shows Sparse Tableau very comparable in speed, but not yet a clear winner.*



# ***Computational Experience with Sparse Tableau Formulation OPF***

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	POLAR- $Y_{bus}$		REC-IV- $Y_{bus}$		REC-STF	
	Obj	Time	Obj	Time	Obj	Time
case118	129660.68	0.3sec	129660.68	0.3sec	129660.68	0.3sec
case300	719725.07	0.6sec	719725.07	2sec	719725.07	0.6sec
case2383wp	1868511.82	5.6sec	1862367.02	5.2sec	1862367.02	6.3sec
case3012wp	2591706.57	5.8sec	2582670.47	5.9sec	2582670.47	6sec
case3120sp	2142703.76	5.8sec	2141532.10	6.3sec	2141532.10	6.5sec
case3375wp	7412030.67	54sec	7404635.99	11.7sec	7404637.15	11.4sec



## ***Take Away Points***

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- *Many parts of power system are, in their hearts, fairly simple circuits, often linear.*
- *Many of the historic “tricks”/reductions in power grid model formulations are arguably becoming less advantageous, because of advances in computational tools, and because new component technologies undermine assumptions needed for these shortcuts.*
- *Standard circuit analysis tools, and in particular Sparse Tableau formulations, facilitate model construction that is versatile, very accessible to the non-specialist, and (in first experiments) computationally just as fast.*