Chance-constrained AC OPF with probabilistic guarantees and scalable performance

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Control of Complex Systems: An Integrated Perspective on Modern Power Grid Control
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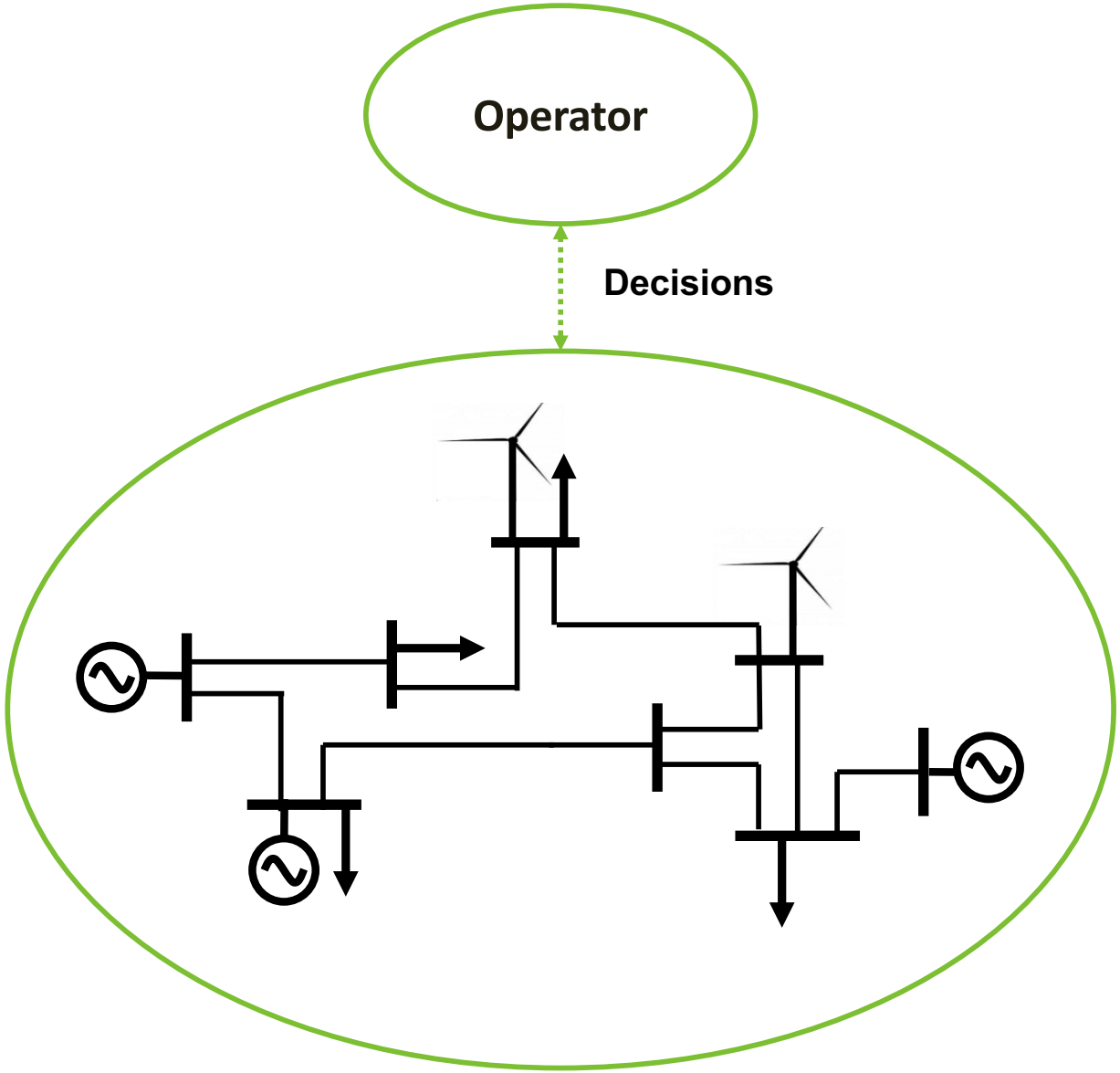
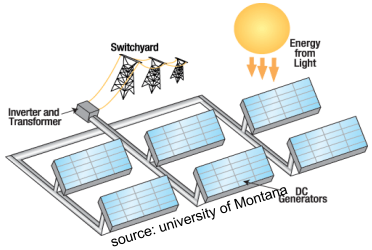
Decisions under uncertainty

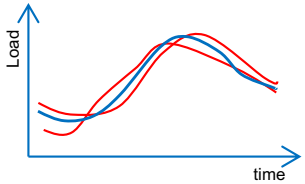


Wind power

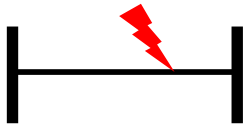
PV power



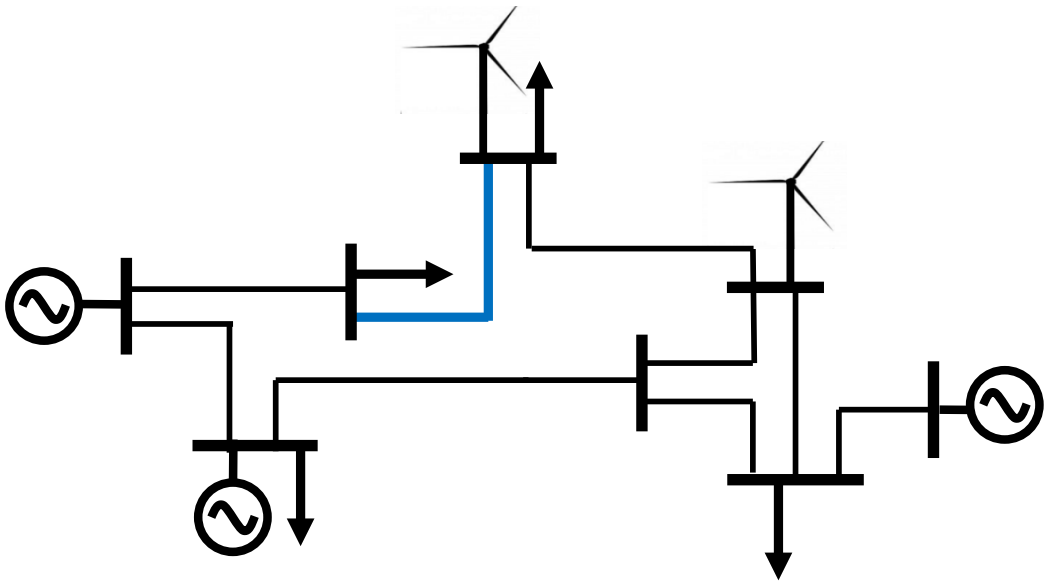
Demand

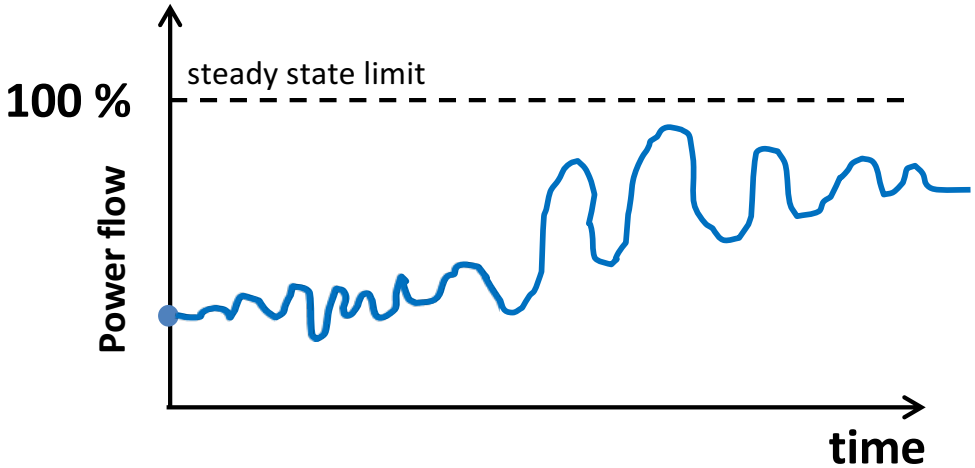


Contingencies



Decisions under uncertainty



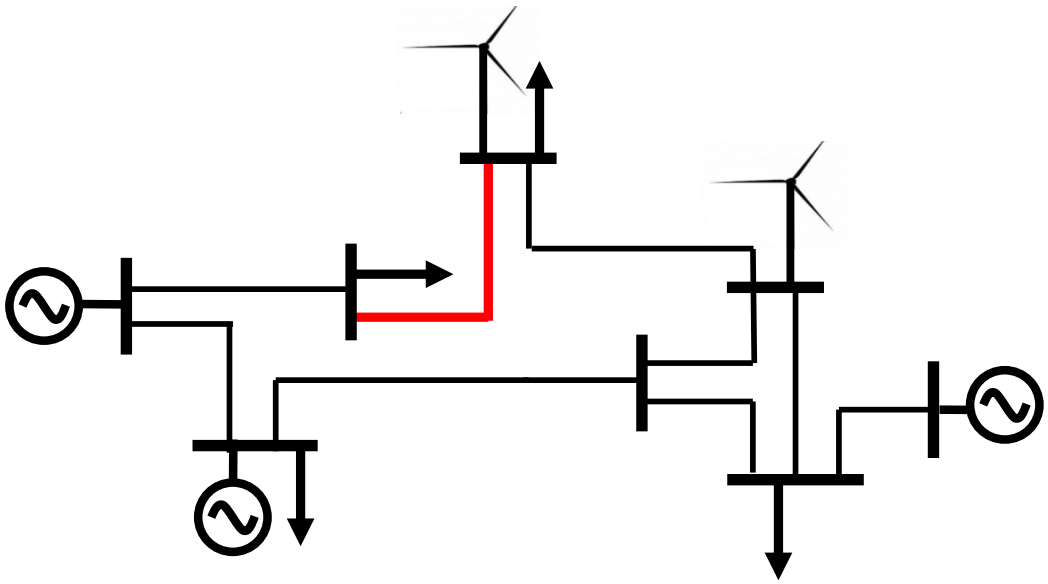


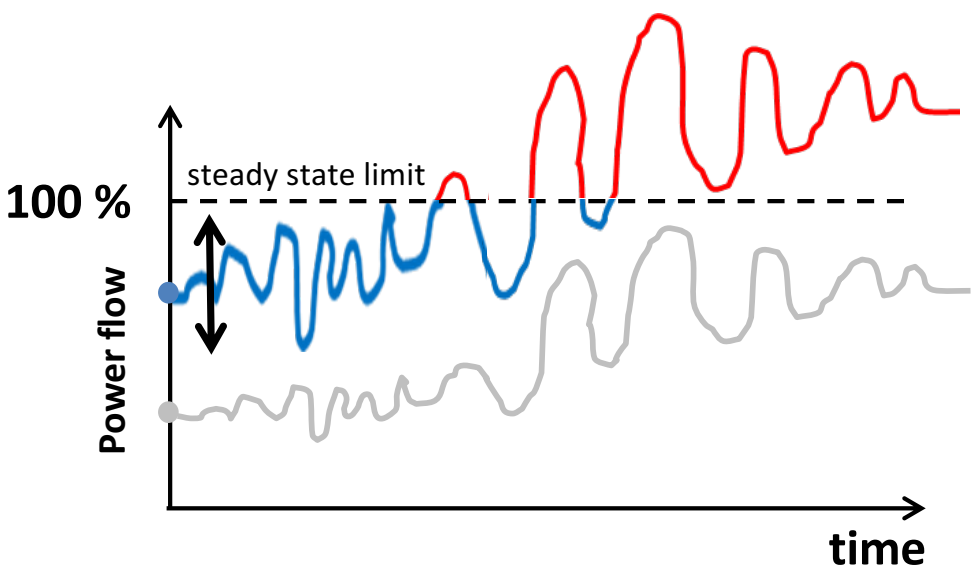
In the past:

- Not heavily loaded system
- Low share of Renewable Energy Sources (RES)

➡ **Enough margin to withstand uncertainty**

Decisions under uncertainty

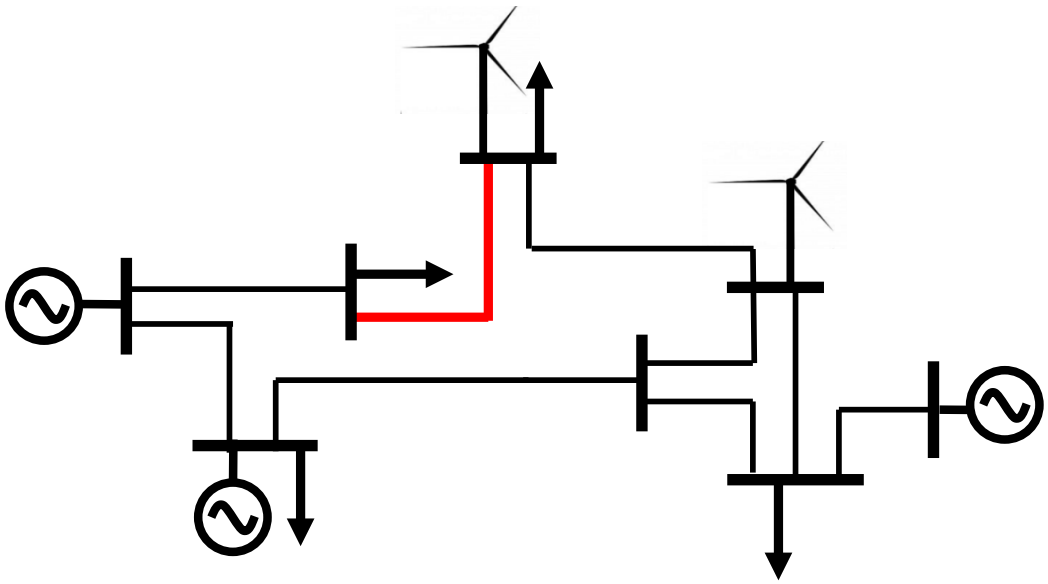


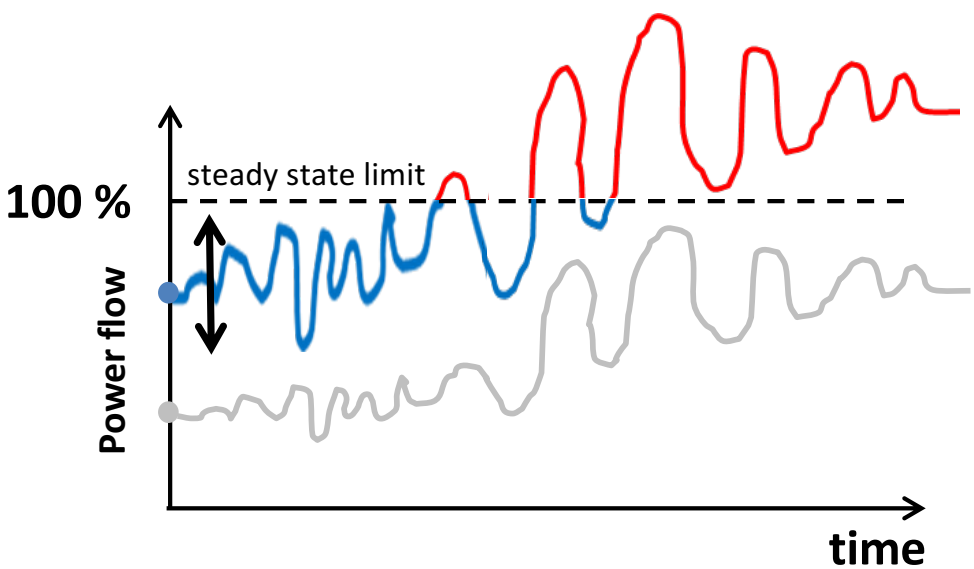


But..

- ⬆ demand
  (higher loading on the system)
- ⬆ penetration of RES
  (higher uncertainty)

Decisions under uncertainty





- Operating closer to the margins
- Maybe not possible to withstand uncertainty

Need for reliable

stochastic Optimal Power Flow formulations

Outline

- The AC Optimal Power Flow (OPF) problem
- Stochastic AC OPF formulation
- Conclusions

Outline

- ❑ **The AC Optimal Power Flow (OPF) problem**
- ❑ **Stochastic AC OPF formulation**
- ❑ **Conclusions**

General AC OPF model

$$\min_x J(x)$$ quadratic cost of generation

subject to

$$g(x, \delta^f) = 0$$ power flow equations

$$h(x, \delta^f) \leq 0$$ operational limits

x voltage magnitude and angle

δ^f wind power forecast

OPF challenges

- **Nonconvex constraints**
- **Need to be applied in large networks**

Different solution methodologies aim to find an approximate solution, a local minimum or ideally the optimal solution in a scalable manner

OPF solution methodology examples

- **Approximations**
  DC OPF: linearized power flow equations - quadratic problem

- **Finding the global optimum**
  Convex Relaxations: Semidefinite program (SDP), Second order cone program (SOCP)

- **Finding a local minimum**
  AC-QP OPF : Quadratic program (QP) formulation - separate AC power flow based update

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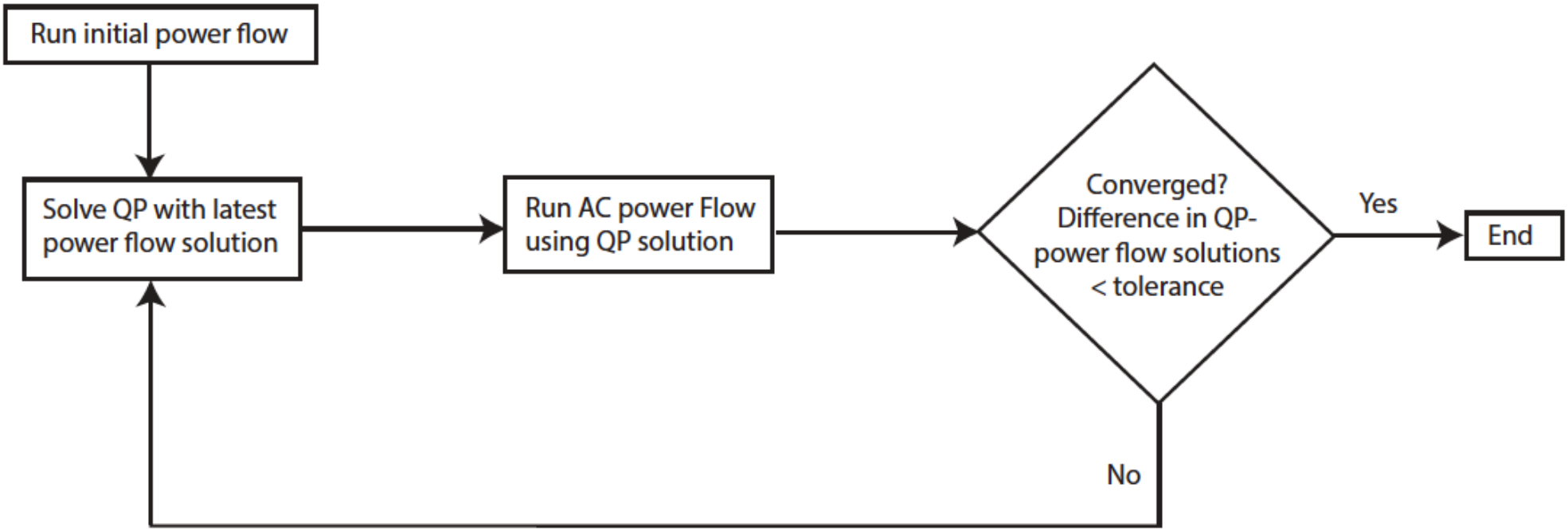
AC-QP OPF algorithm



- Initialize the AC power flow using an SCOP solution
- SOCP provides a lower bound of the OPF problem

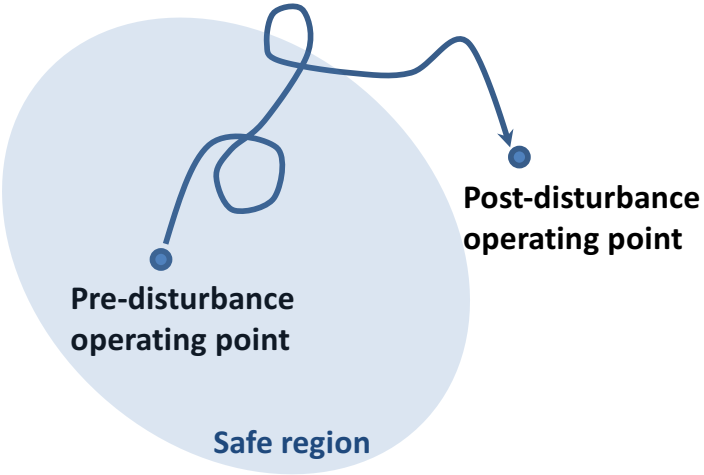
[A. Wood, B. Wollenberg, and G. Sheble, Power Generation, Operation and Control, 3rd ed. John Wiley and Sons, Inc., 2013]

Outline

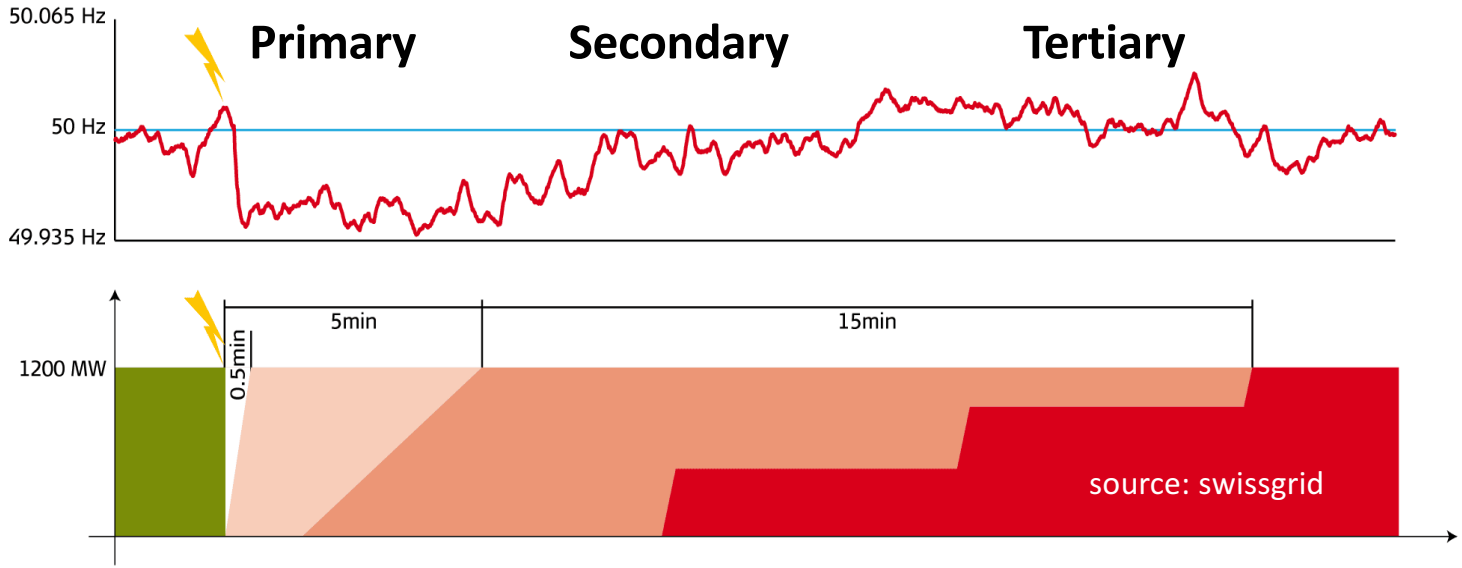
- The AC Optimal Power Flow (OPF) problem
- **Stochastic AC OPF formulation**
- Conclusions

Steady state operating points

Given a generation-load mismatch the automatic control loops will lead to a new post-disturbance operating point



Example: Frequency control

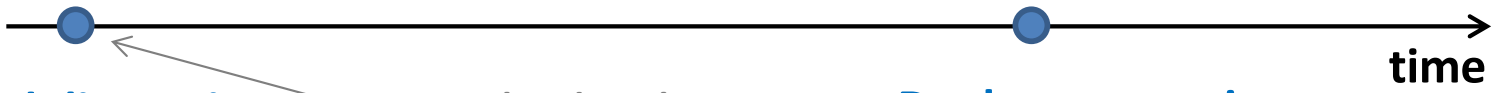


Steady state operating points

Satisfy the constraints at the steady state point after

- **Primary control** $P_G^p = P_G^f - d_p \, \Delta P_w$
- **AGC response (secondary control)** $P_G^s = P_G^f - d_s \, \Delta P_w$
- **a redispatch action (or tertiary control)** $P_G^r(\Delta P_w)$

Optimal decision making under uncertainty



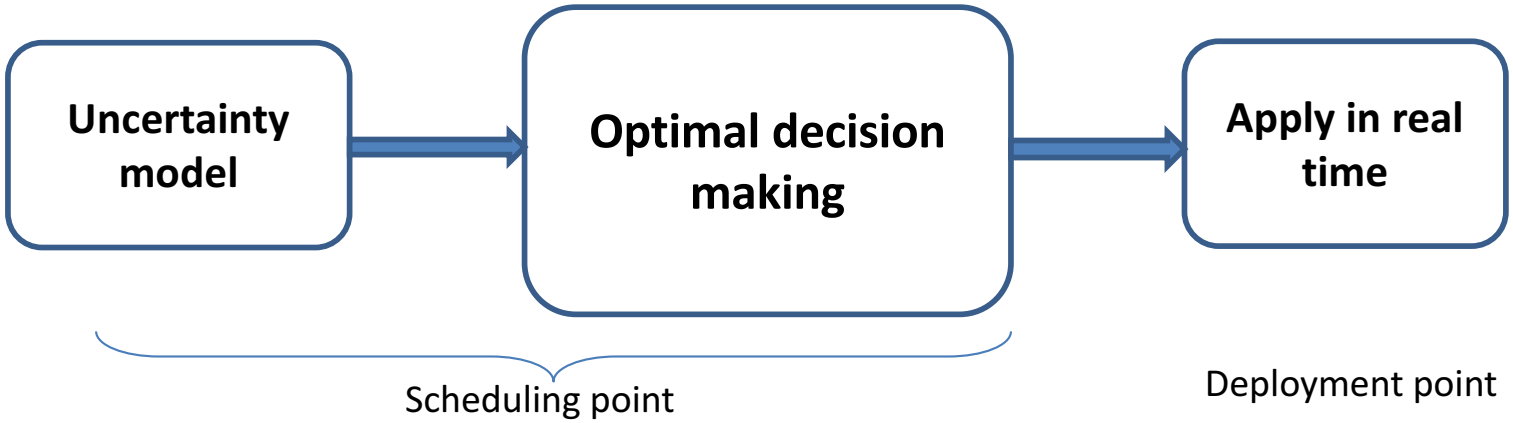
Scheduling point

- **Optimal decisions**
- **Uncertain forecast**
- **Historical data of the uncertainty**

Day ahead market: one day before
Intra-day market hours before

Deployment point

- **Apply the optimal decisions**
- **Realization of the uncertainty**



Chance Constrained OPF

$$\min_x J(x)$$

subject to

$$g(x, \delta^f) = 0$$
$$h(x, \delta^f) \leq 0$$

$$\min_x J(x)$$

subject to

$$\mathbb{P}\big(\delta \in \mathbb{R}^{N_w} \mid g(x,\delta) = 0, \; h(x,\delta) \leq 0\big) \geq 1 - \varepsilon$$

Satisfy the constraints in a probabilistic sense

Chance Constrained OPF

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subject to

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Solve the chance constraint using data-based optimization

Data-based optimization

The scenario approach [1]

- No assumptions on the underlying distribution of the uncertainty
- Guarantees a-priori that the chance constraint will be satisfied with a certain confidence
- Requires convexity with respect to decisions variables

Probabilistically robust design [2]

- Mixture of randomized and robust optimization
- No assumptions on the underlying distribution of the uncertainty
- Guarantees a-priori that the chance constraint will be satisfied with a certain confidence
- No particular structure required with respect to the decision variable

The non-convex scenario optimization [3]

- No assumptions on the underlying distribution of the uncertainty
- Applicable to con-convex problems
- Provides a-posteriori guarantees

[1] G. Calafiore and M. Campi, TAC, 2006
[2] K. Margellos, P. Goulart and J. Lygeros, TAC, 2014
[3] M. C. Campi, S. Garatti, F. A. Ramponi, *CDC,* 2015

Data-based optimization

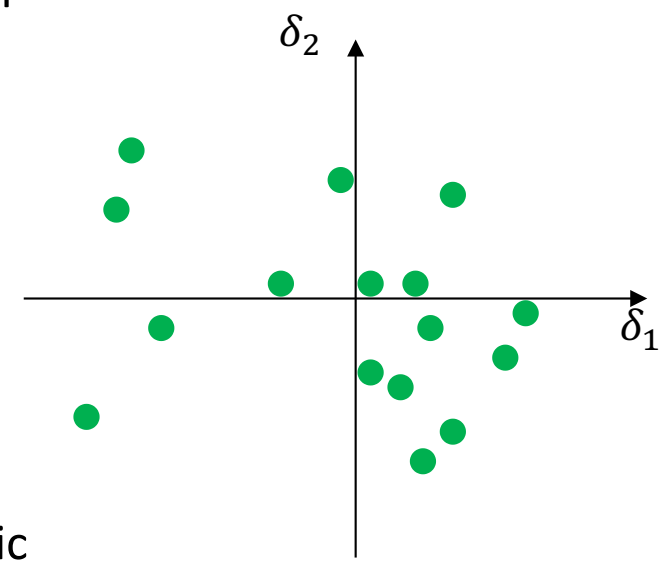
The scenario approach [1]

- Substitute the chance constraint with a finite number of hard constraints based on scenarios of the uncertainty

$$\mathbb{P}\left(\delta \in \mathbb{R}^{N_w} \mid h(x,\delta) \leq 0\right) \geq 1 - \varepsilon$$

$$\Downarrow$$

$$h(x,\delta_i) \leq 0 \quad \text{for } i = 1 \ldots N$$



- How many scenarios do we need to provide probabilistic guarantees?

$$N \geq \frac{1}{\varepsilon}\frac{e}{e-1}\left(\ln\frac{1}{\beta} + N_x - 1\right)$$

number of scenarios

violation level

confidence

number of decision elements

Data-based optimization

Probabilistically robust design (two-step approach) [2]

- **Step 1**

- Use **the scenario approach** to find '**bounds**' of the uncertainty elements
- How many scenarios do we need to provide probabilistic guarantees?

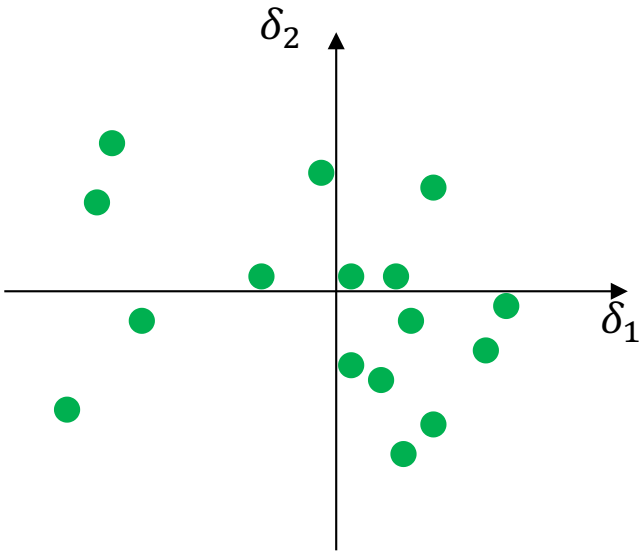
$$N \geq \frac{1}{\varepsilon}\frac{e}{e-1}\left(\ln\frac{1}{\beta} + 2N_\delta - 1\right)$$

number of scenarios

violation level

confidence

number of uncertainty elements



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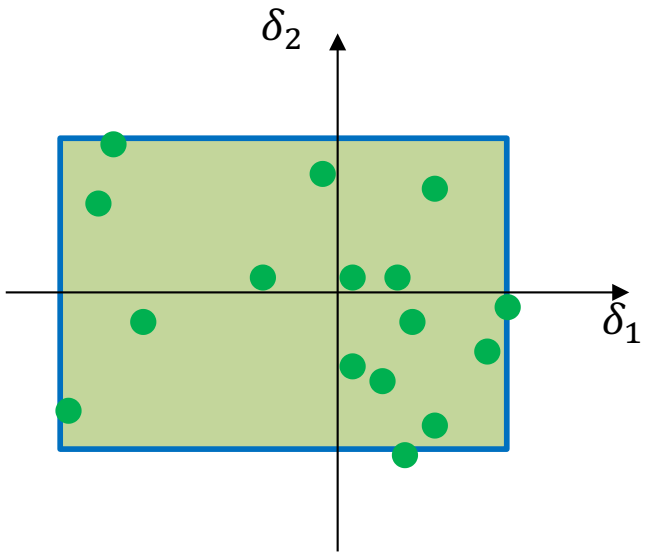


Dealing with the chance constraint

Probabilistically robust design (two-step approach) [2]

- **Step 2**

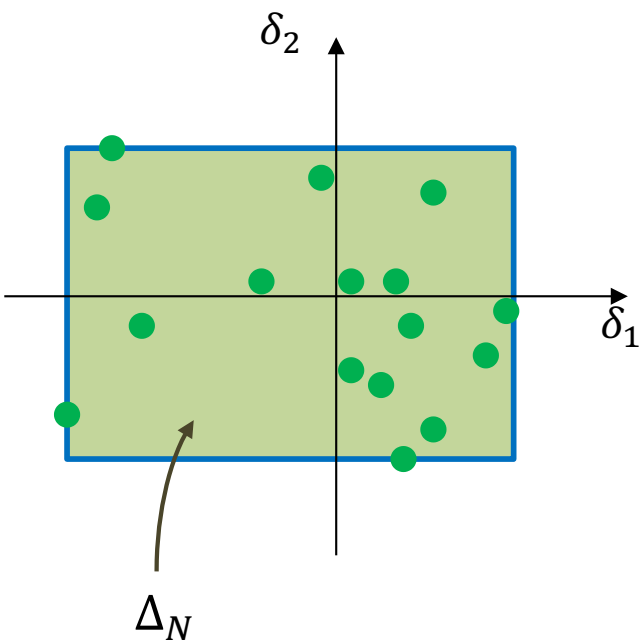
  - Solve a **robust reformulation** of the initial chance constrained problem

  - Any solution of the robust problem is feasible for the chance constrained with confidence at least $1-\beta$

$$\mathbb{P}\big(\delta \in \mathbb{R}^{N_w} \mid h(x,\delta) \leq 0\big) \geq 1-\varepsilon$$

$$\Downarrow$$

$$h(x,\delta) \leq 0 \quad \text{for all} \quad \delta \in \Delta_N$$



Prior work

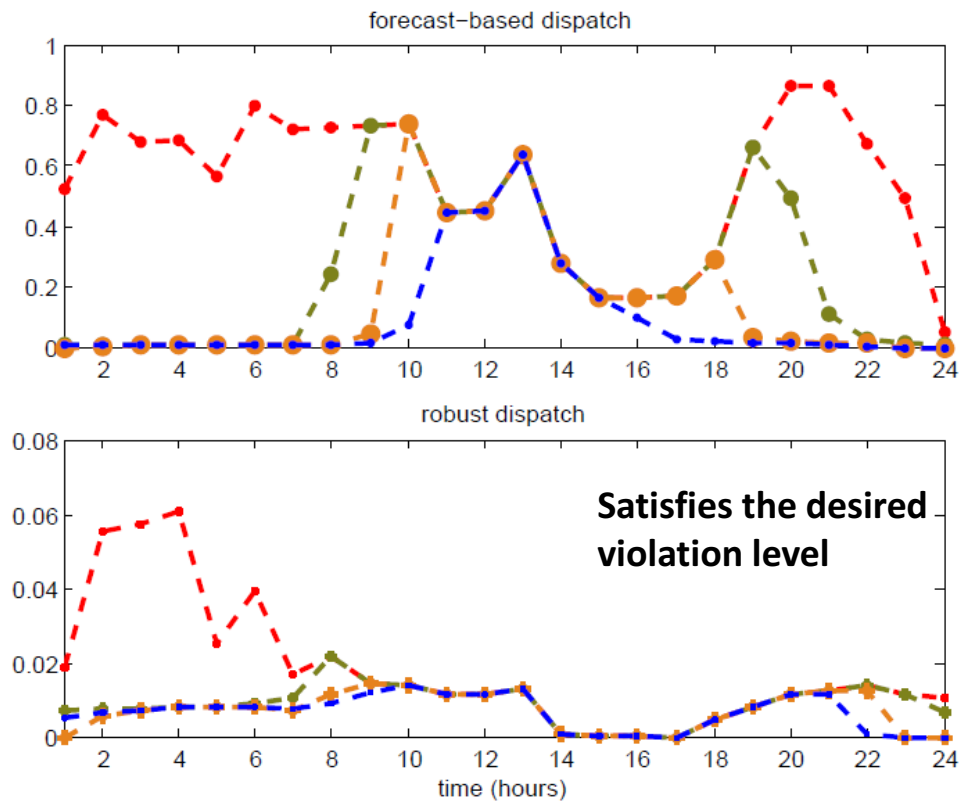
DC power flow

- Using active power policies $P_G^s = P_G^f - d_s \; \Delta P_w$ allow to trivially satisfy the equality constraints and the problem can be tractable
- Methods [1] and [2] were efficiently used
- But solution not accurate for the AC constraints

IEEE 30-bus network

- 4 load profiles
- Desired violation level 10%
- Monte Carlo evaluation for 10000 wind power realizations

Fails to satisfy the desired violation level



[M. Vrakopoulou, K. Margellos, J. Lygeros, and G. Andersson, "Probabilistic guarantees for the N-1 security of systems with wind power generation," in PMAPS 2012]

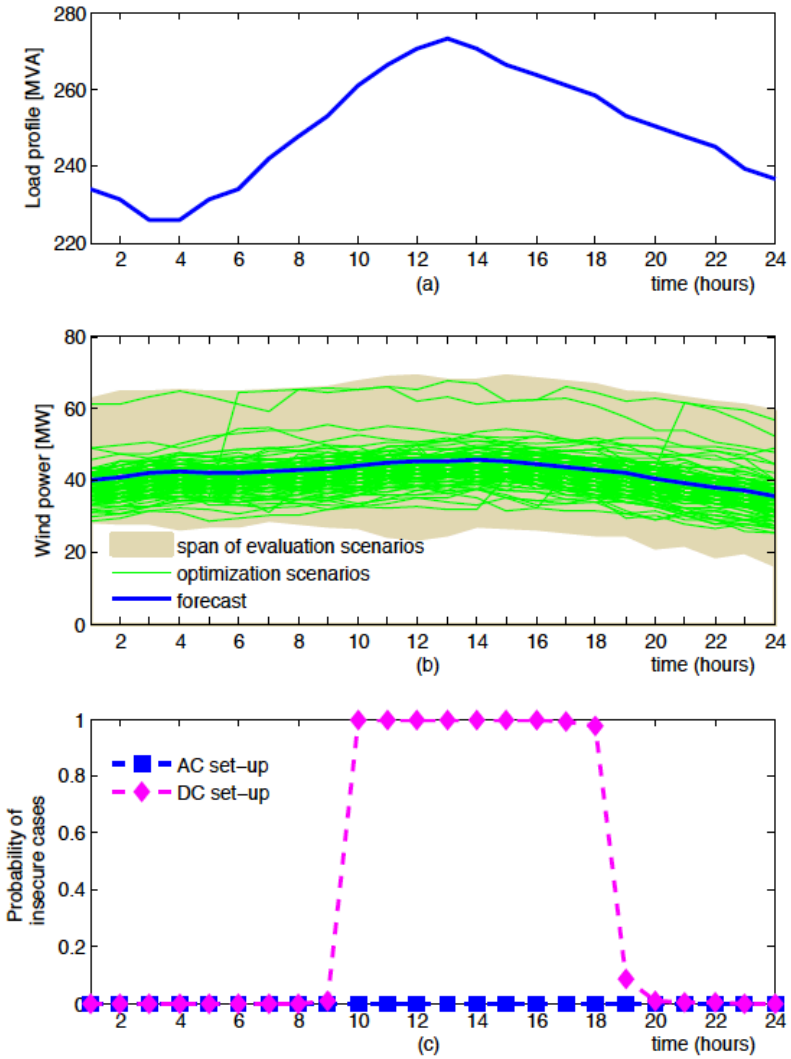
Prior work

AC power flow

- SDP formulation, active power and generator voltage policies need to be used to make the problem tractable.
- Method [2] was applied
- But scalability issues!!

IEEE 14-bus network

- Desired violation level 10%
- Monte Carlo evaluation for 10000 wind power realizations



[M. Vrakopoulou, M. Katsampani, K. Margellos, J. Lygeros, and G. Andersson "Probabilistic security-constrained AC optimal power flow," in IEEE PowerTech Conference, 2013]

Data-based optimization

The non-convex scenario optimization [3]

- Use N scenarios as an input to the non-convex optimization algorithm
- Calculate the *support scenarios* k of the obtained solution
- Provides a-posteriori guarantees for the upper bound of chance constraint violation level ϵ

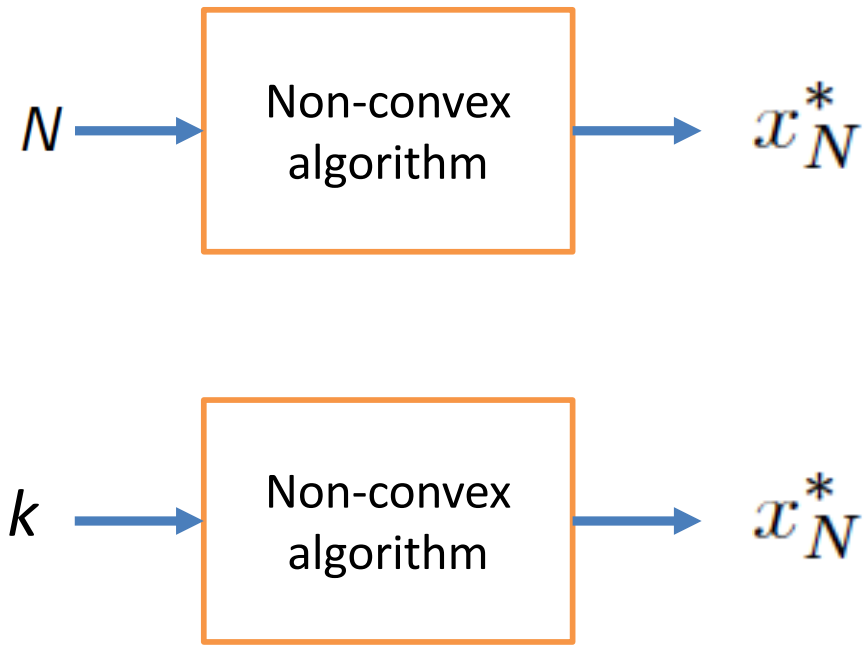
$$\epsilon(k) = \begin{cases} 1, & \text{if } k = N \\ 1 - \sqrt[N-k]{\frac{\beta}{N\binom{N}{k}}}, & \text{otherwise} \end{cases}$$

$\beta \in (0, 1)$, a design parameter representing the probability that the upper bound ϵ will be violated

[3] M. C. Campi, S. Garatti, F. A. Ramponi, *CDC,* 2015

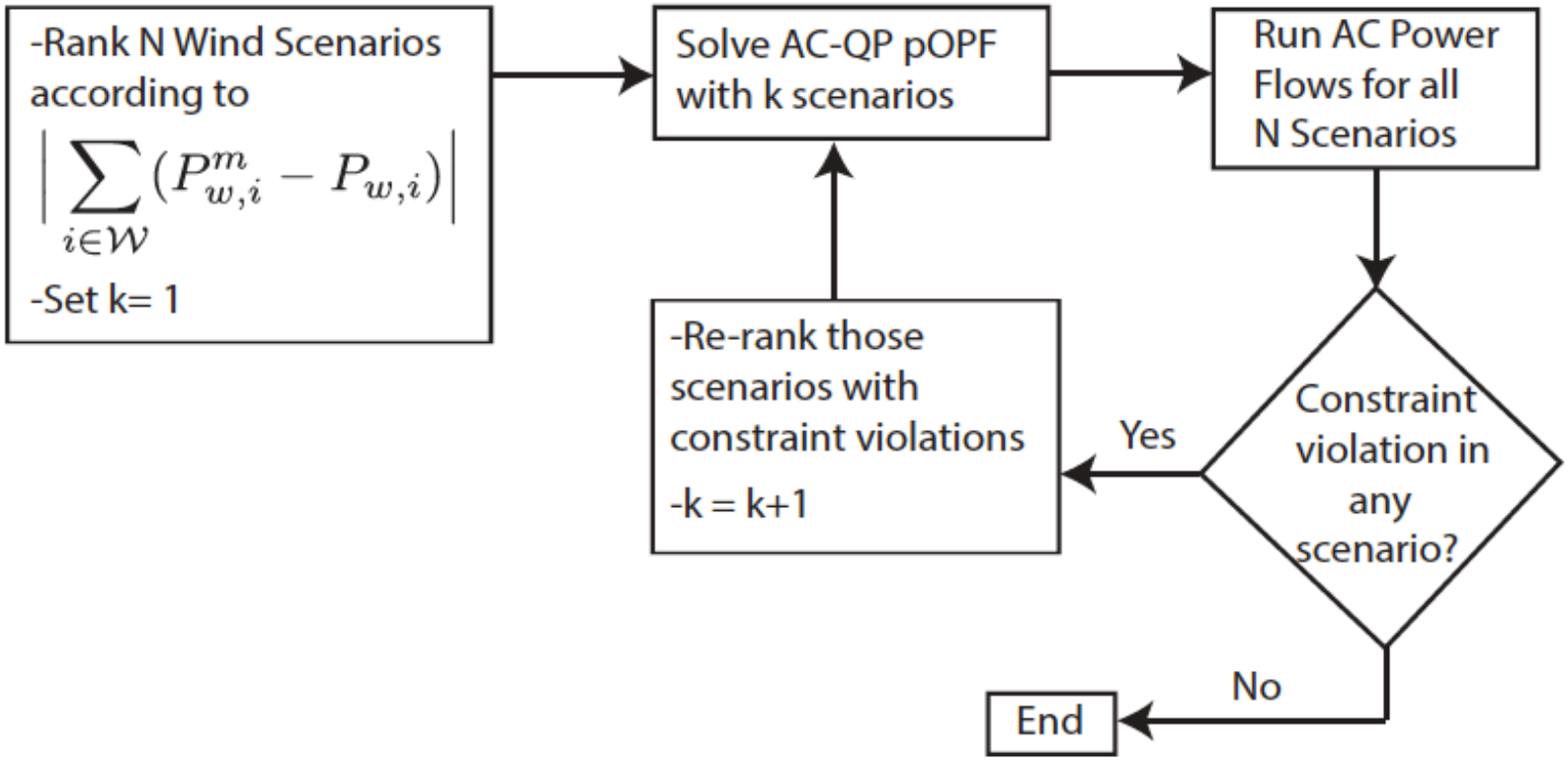
Data-based optimization

Support scenarios



Can we find a small number of scenarios k that must be included in the problem so that the solution would be the same as the one using all the N scenarios?

Stochastic AC-QP OPF



[J. F. Marley, M. Vrakopoulou and I. A. Hiskens, An AC-QP optimal power flow algorithm considering wind forecast uncertainty, IEEE PES Innovative Smart Grid Technologies, ISGT-Asia 2016]

Scenarios in the OPF

Given a set of possible wind scenarios, $\mathcal{S}$:

$\min C(P_g) \leftarrow$ (Quadratic) Cost of Conventional Generation

subject to

$g(x) \leq 0 \leftarrow$ Operational Limits & Line Flow Constraints

$h(x) = 0 \leftarrow$ (AC) Power Flow Equations

$\forall m \in \mathcal{S}$:

$g(x, x^m) \leq 0 \leftarrow$ Operational Limits & Line Flow Constraints

$h(x, x^m) = 0 \leftarrow$ (AC) Power Flow Equations

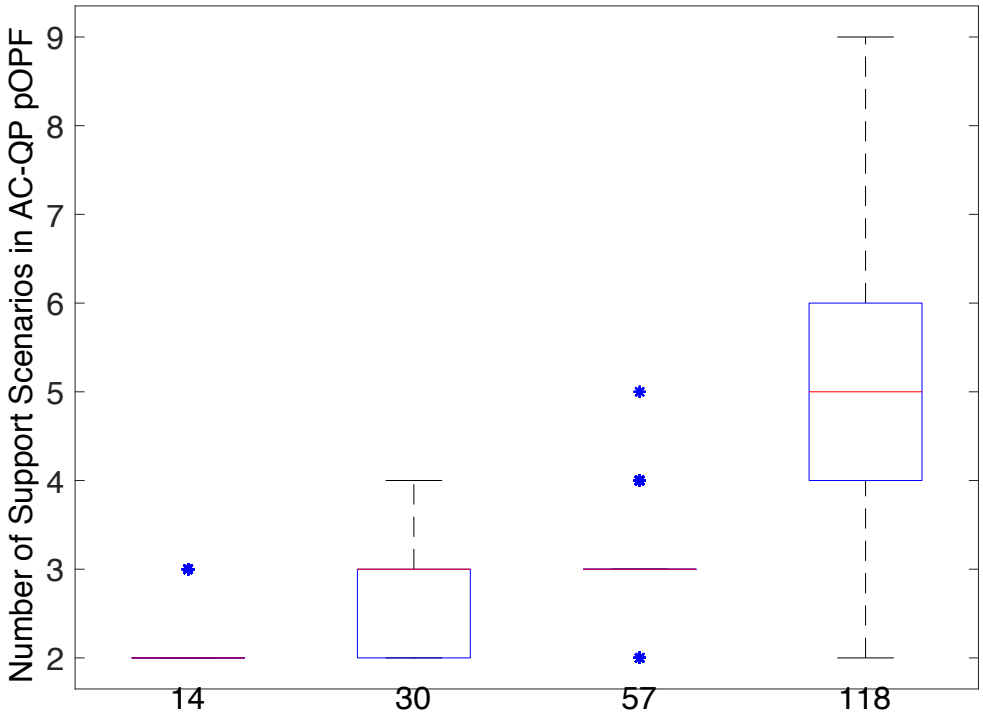
Case studies

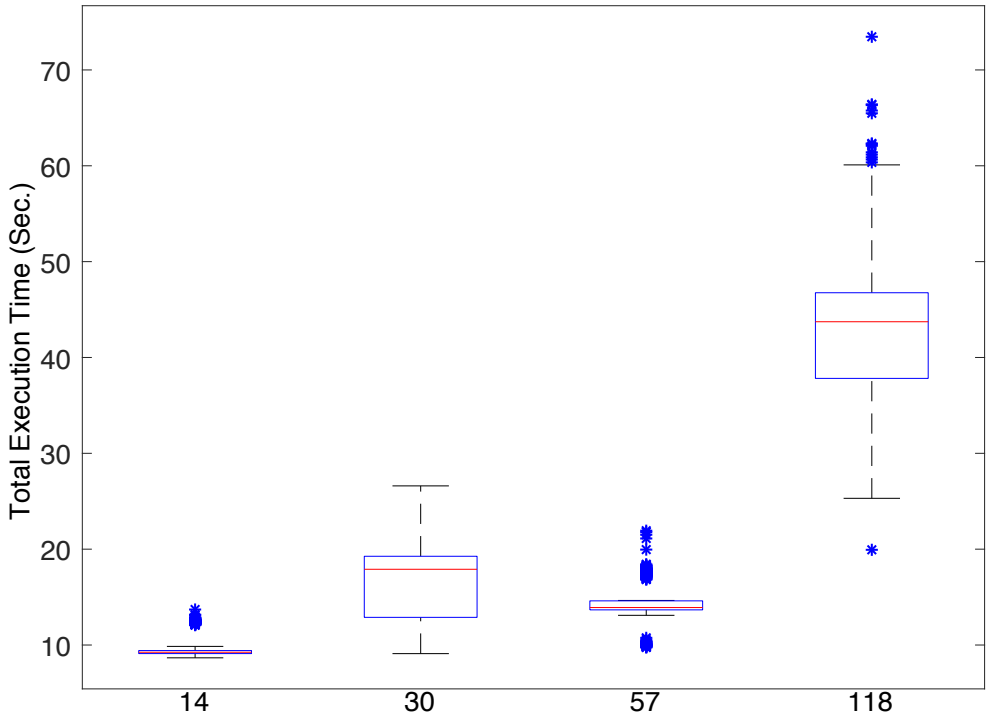
Test the stochastic AC-QP algorithm for the following IEEE benchmark networks:

- 14-Bus: 5 Generators, 20 Branches, 2 wind buses
- 30-Bus: 6 Generators, 41 Branches, 5 wind buses
- 57-Bus: 7 Generators, 80 Branches, 7 wind buses
- 118-Bus: 54 Generators, 186 Branches, 10 wind buses

N=1500 scenarios
β=0.001

Increasing the network size





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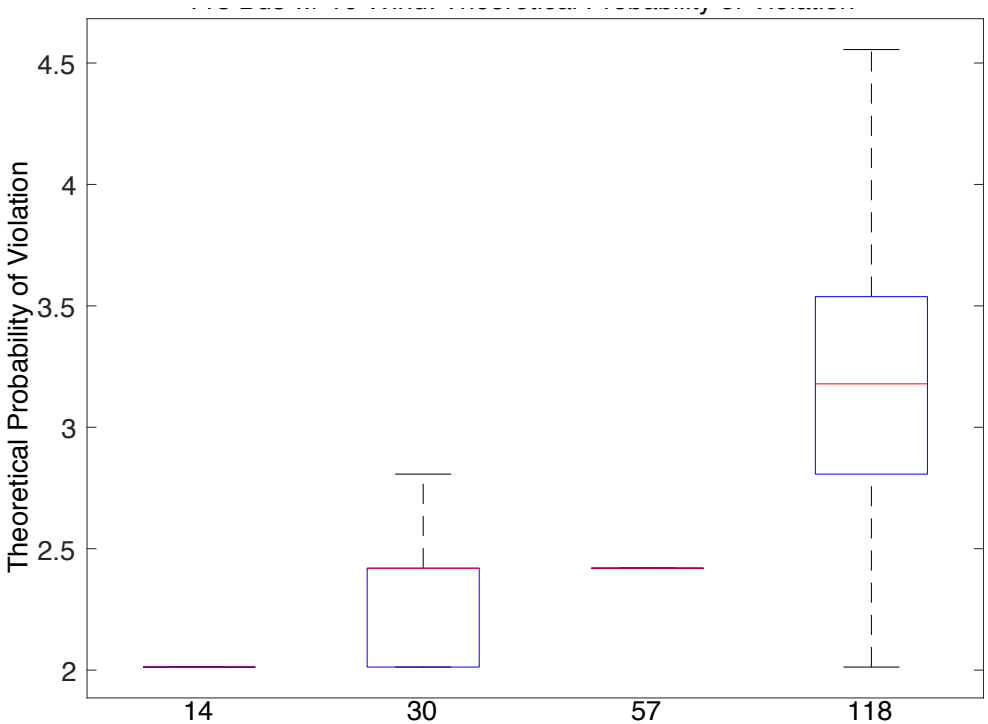
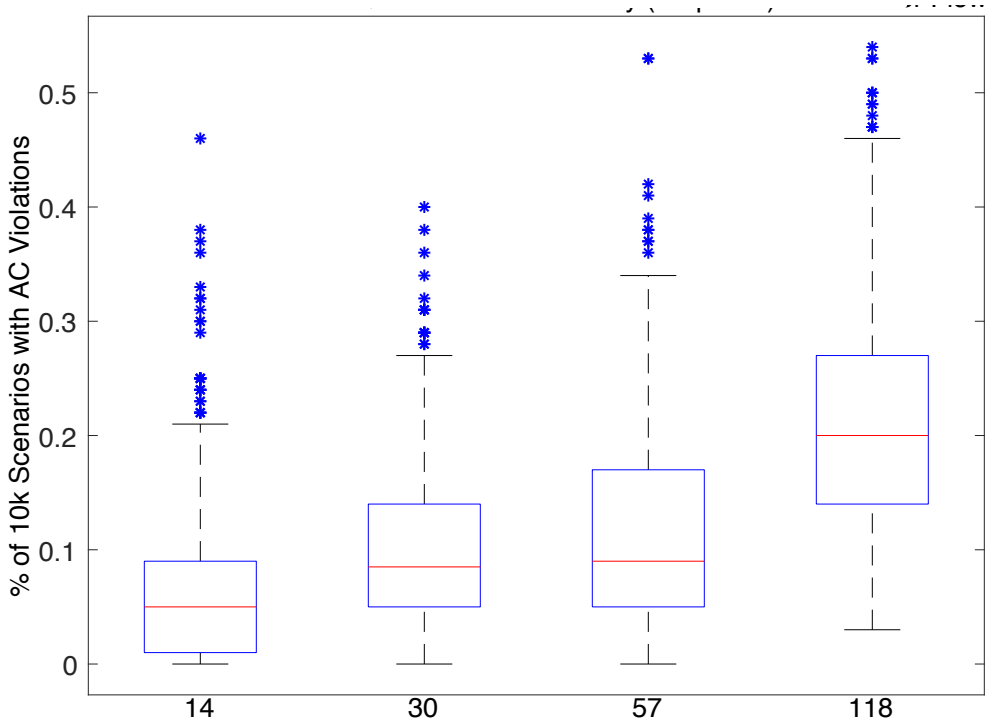
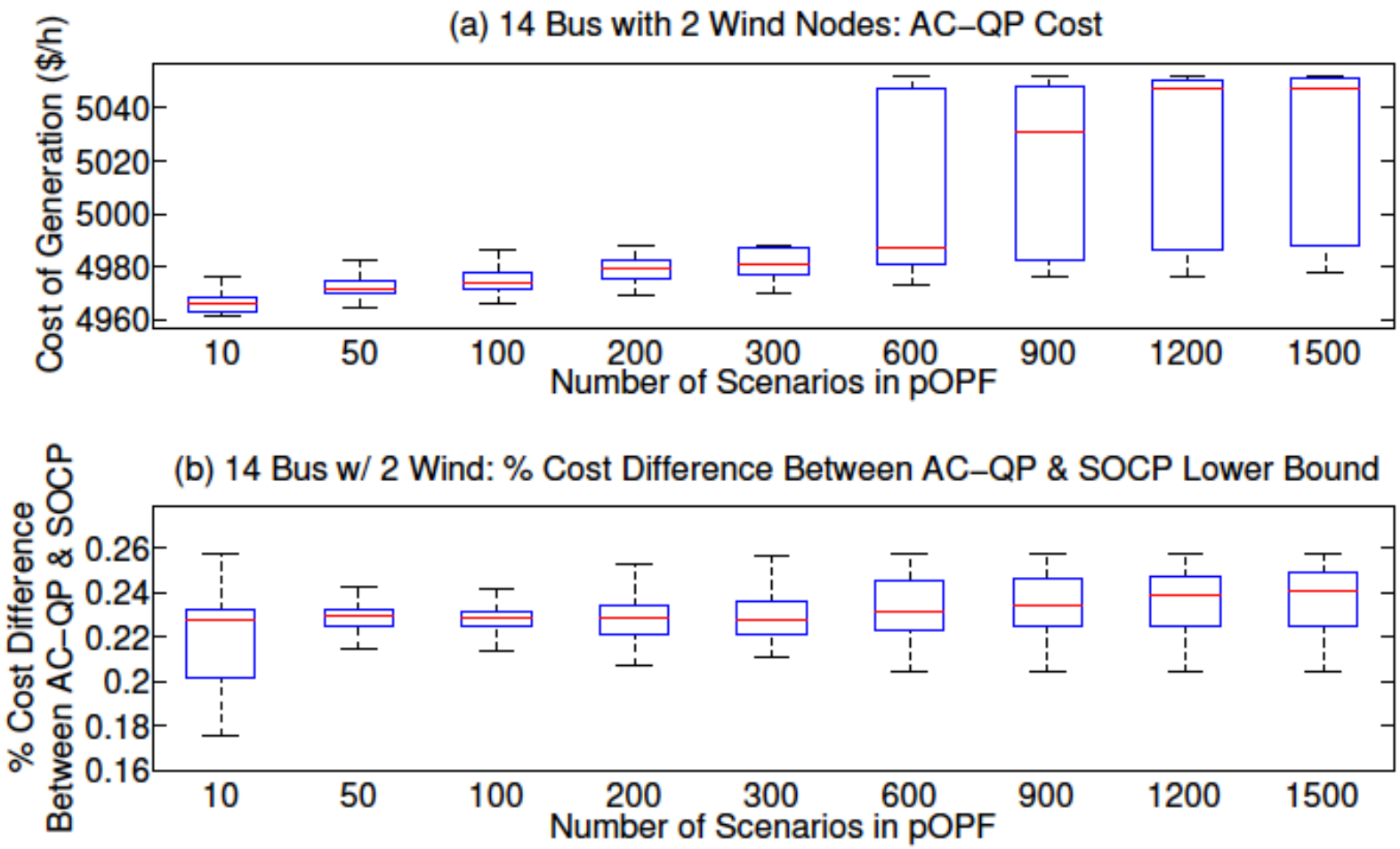




Increasing the number of scenarios N



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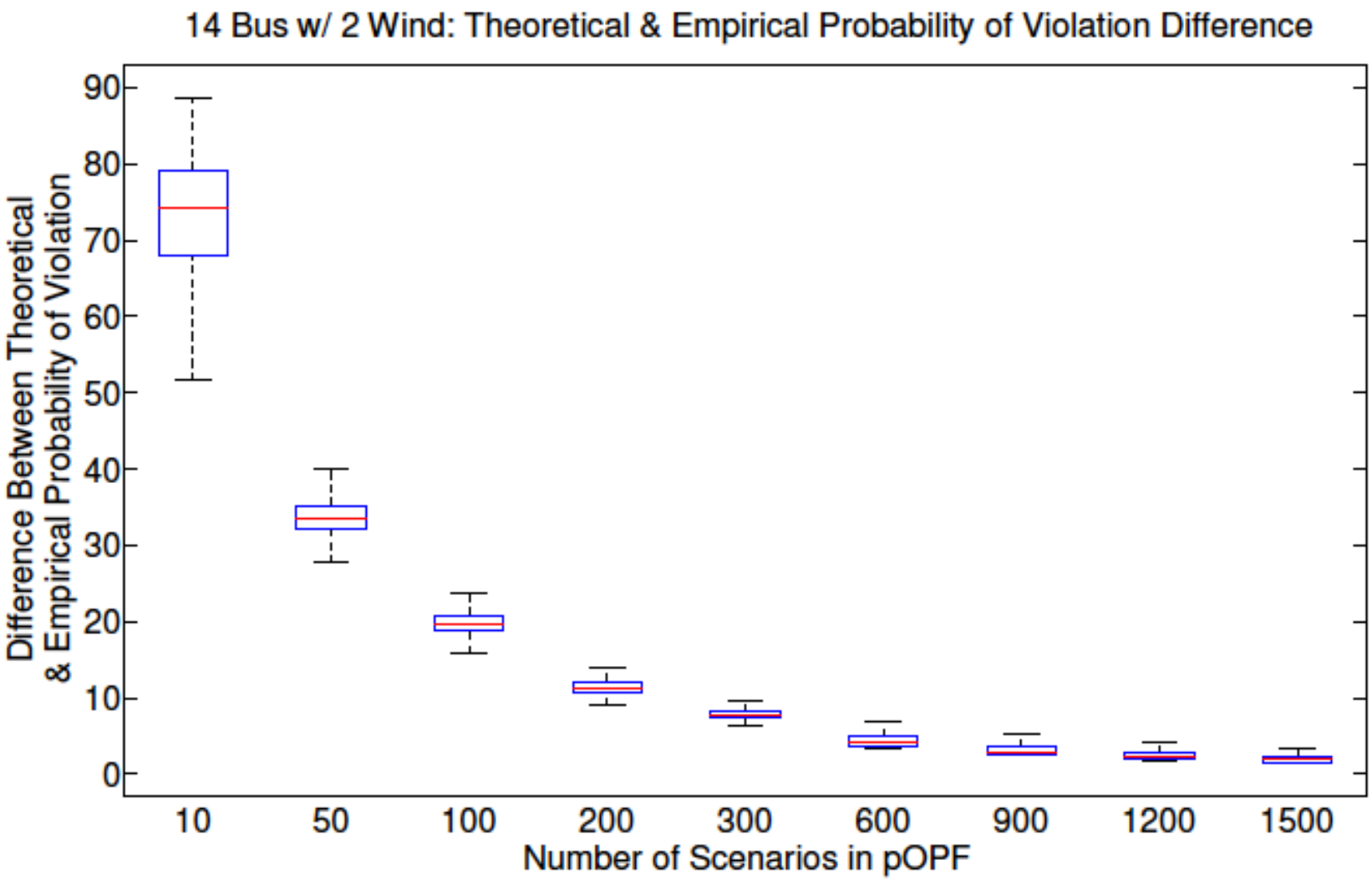

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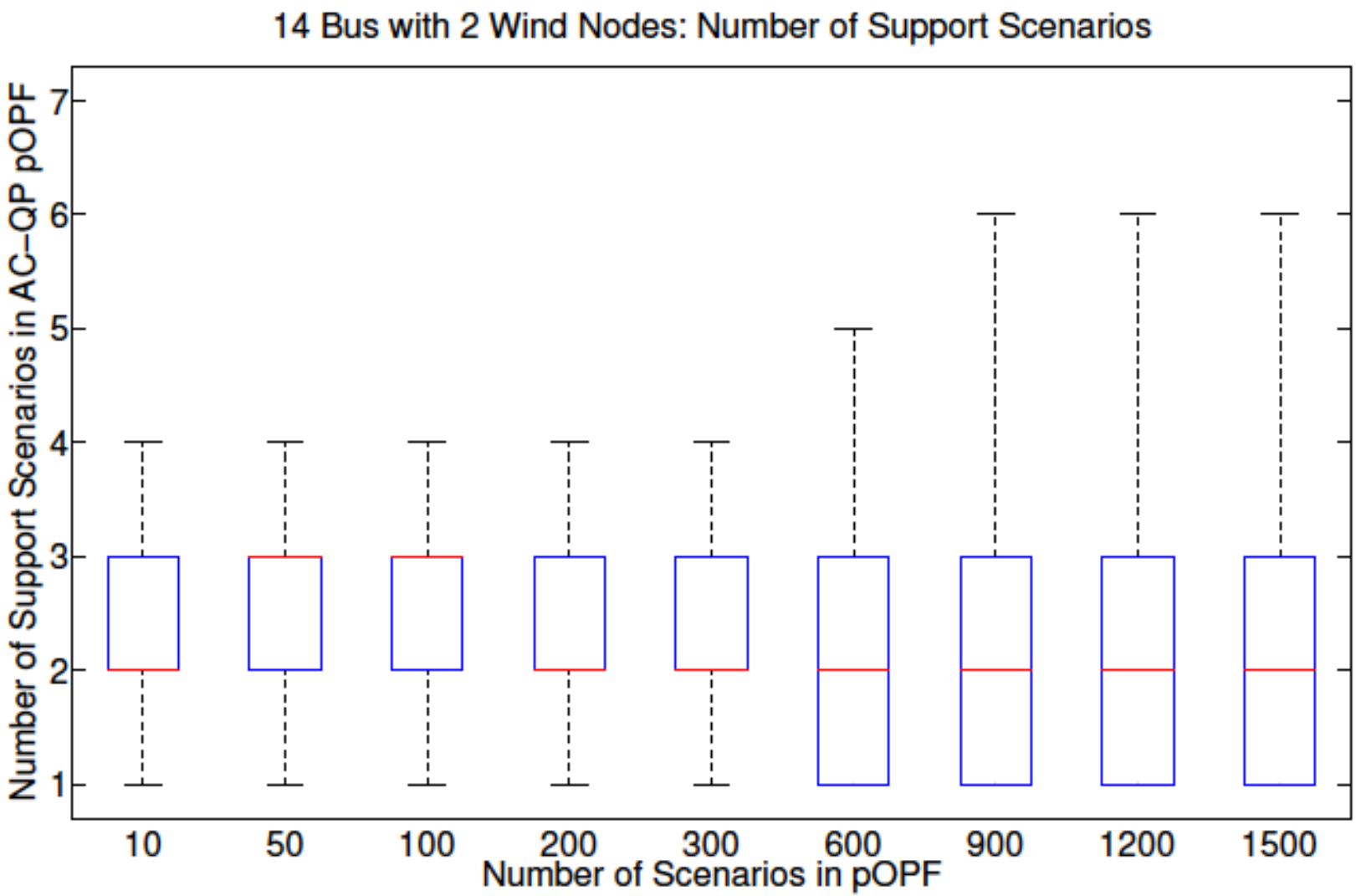

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- The AC Optimal Power Flow (OPF) problem
- Stochastic AC OPF formulation
- **Conclusions**

Conclusions

The AC-QP OPF method has been extended to include wind power uncertainty, through the addition of a finite number of wind scenarios.

The resulting stochastic AC-QP OPF algorithm offers several advantages:

- It does not rely upon model approximations and produces an AC feasible solution.
- It provides a probabilistically robust solution with a-posteriori probabilistic violation guarantees.
- The number of support scenarios remain small as the problem size increases highlighting promising scalability properties

Thank you for your attention!

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