

# Wholesale electricity market design under uncertainty



**Subhonmesh Bose**

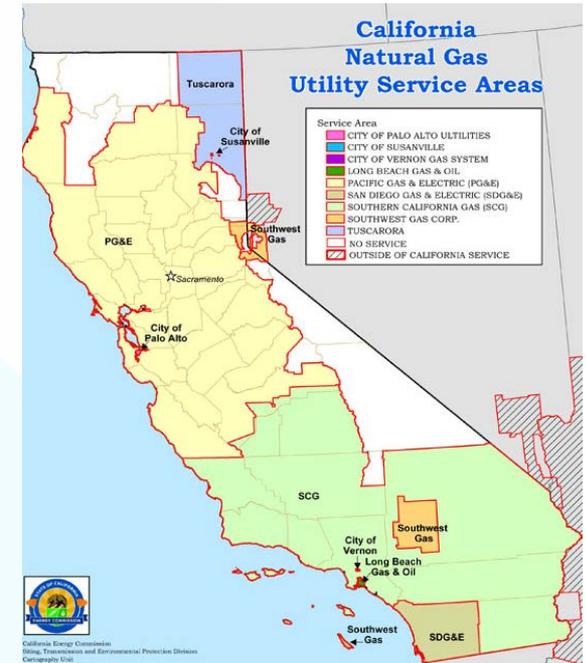
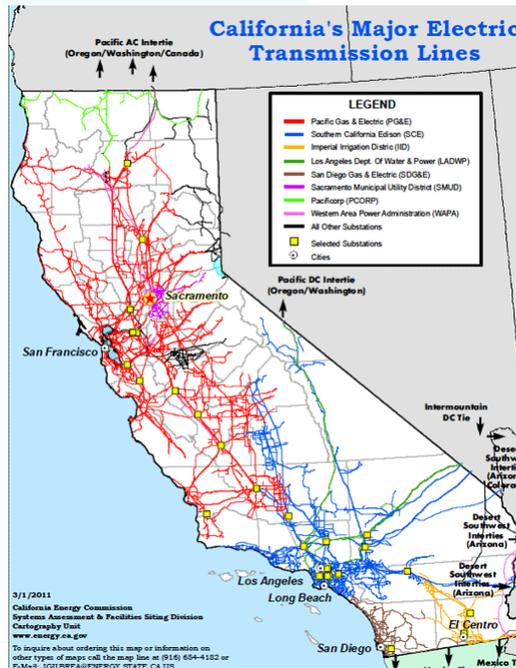
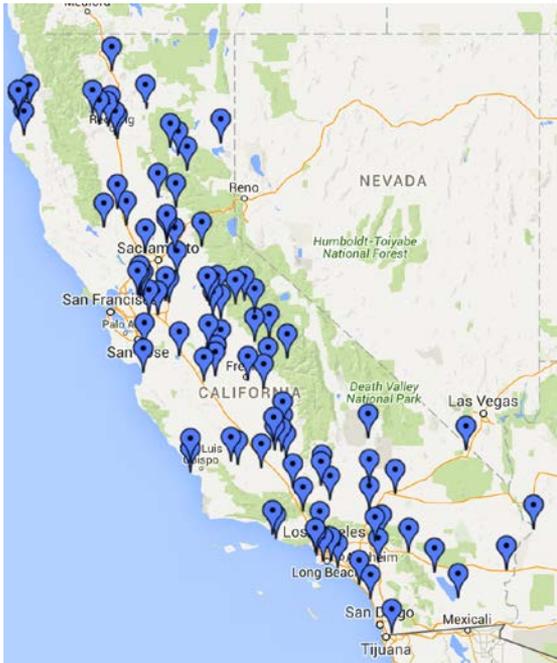
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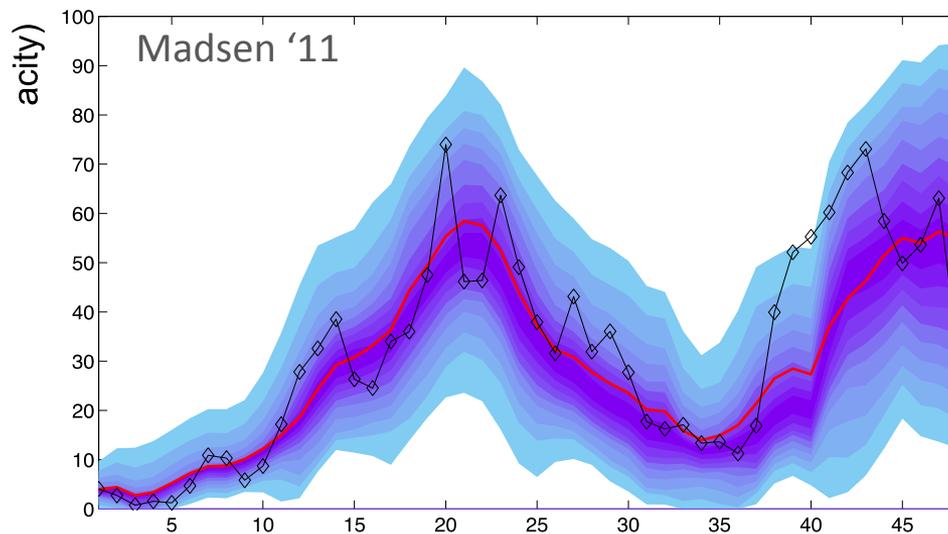
# Wholesale electricity markets



California ISO  
Your Link to Power



# Do market structures need to change with deepening penetration of variable renewable supply?



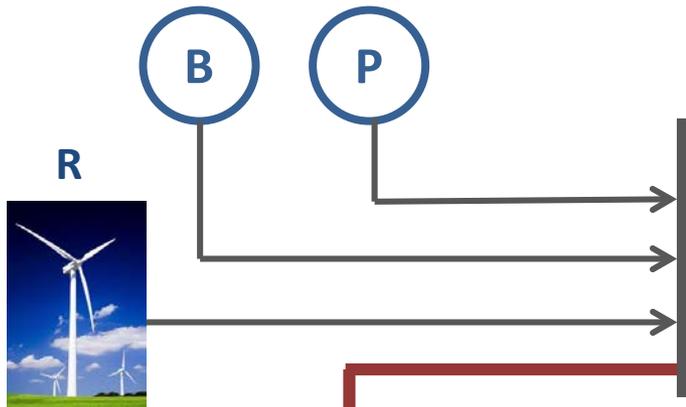
*Uncertain*  
*Intermittent*  
*Non-dispatchable*

**Q1.** *Do we need to change the forward market design?*

**Q2.** *How can we design instruments to mitigate financial risks?*

# A copperplate power system

$\Omega =$  Scenarios of available renewable supply

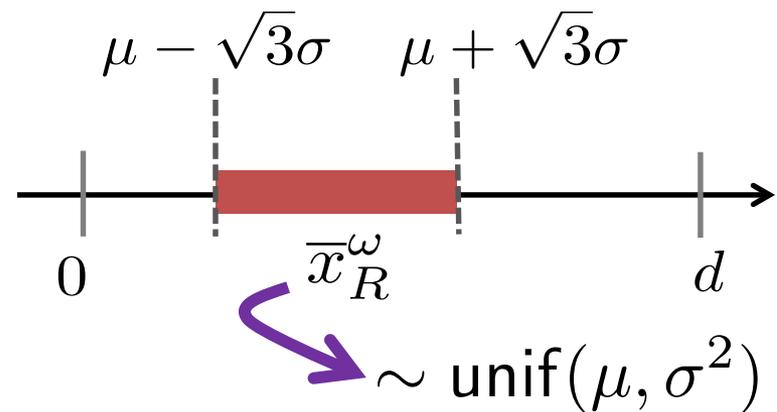


$d > 0$

Uncontrollable but  
predictable demand



Renewable power producer

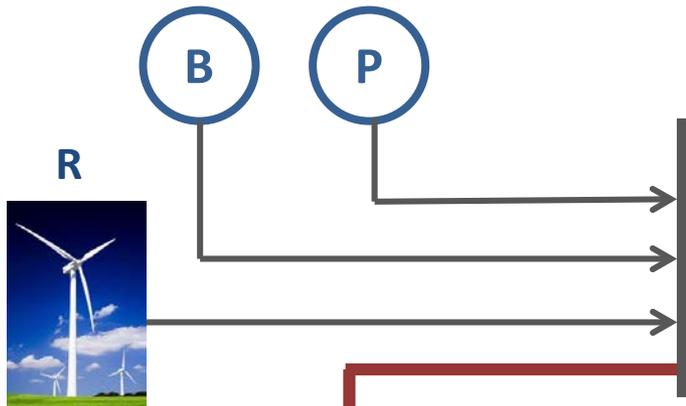


$$0 \leq x_R^\omega \leq \bar{x}_R^\omega$$

**cost** = 0.

# A copperplate power system

$\Omega =$  Scenarios of available renewable supply



Uncontrollable but predictable demand

**B**

**Baseload generator**

$$0 \leq x_B^\omega \leq \infty$$

$$|x_B^\omega - X_B| = 0$$

$$\text{cost} = x_B^\omega$$

*Forward set point*

$$\rho > 1$$

**P**

**Peaker power plant**

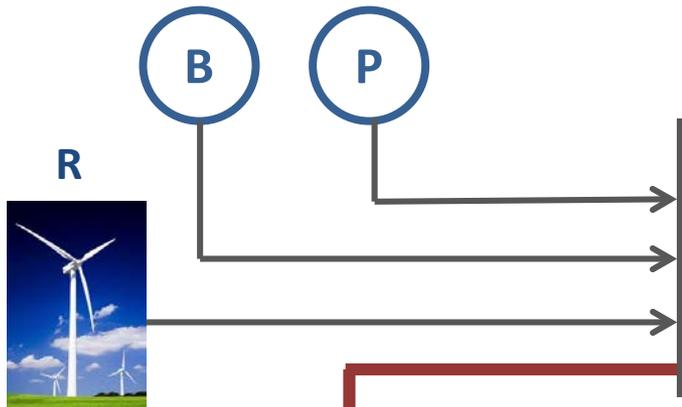
$$0 \leq x_P^\omega \leq \infty$$

$$|x_P^\omega - X_P| \leq \infty$$

$$\text{cost} = \rho x_P^\omega$$

# A copperplate power system

## *A certainty-equivalent based forward dispatch*



$d > 0$

Uncontrollable but  
predictable demand

$$\begin{aligned} & \text{minimize}_{X_B, X_P, X_R} && 1 \cdot X_B + \rho \cdot X_P + 0 \cdot X_R, \\ & \text{subject to} && X_B + X_P + X_R = d, \\ & && X_B \geq 0, X_P \geq 0, \\ & && 0 \leq X_R \leq \mu. \end{aligned}$$

Forward price = optimal Lagrange multiplier  $P^*$

Summary of forward market clearing:

$$X_B^* = d - \mu,$$

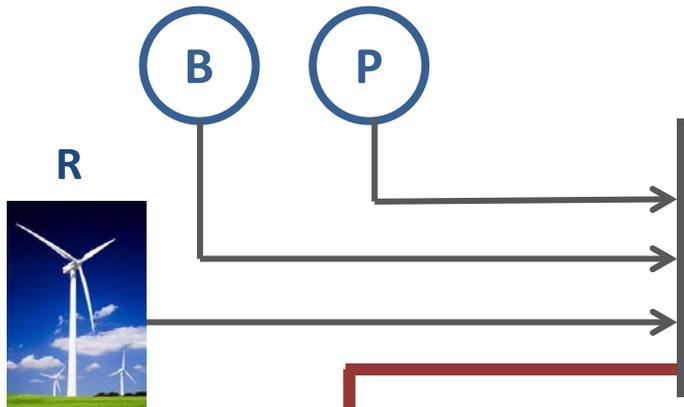
$$X_P^* = 0,$$

$$X_R^* = \mu,$$

$$P^* = 1.$$

# A copperplate power system

*Real-time balancing,  
given forward dispatch*



minimize  
 $x_B^\omega, x_P^\omega, x_R^\omega$

subject to

$$1 \cdot x_B^\omega + \rho \cdot x_P^\omega + 0 \cdot x_R^\omega,$$

$$x_B^\omega + x_P^\omega + x_R^\omega = d,$$

$$|x_B^\omega - X_B| = 0,$$

$$x_P^\omega \geq 0,$$

$$0 \leq x_R^\omega \leq \bar{x}_R^\omega.$$

Summary of real-time market clearing:

$$x_B^{\square,!} = d - \mu,$$

$$x_P^{\square,!} = (\mu - !)^+,$$

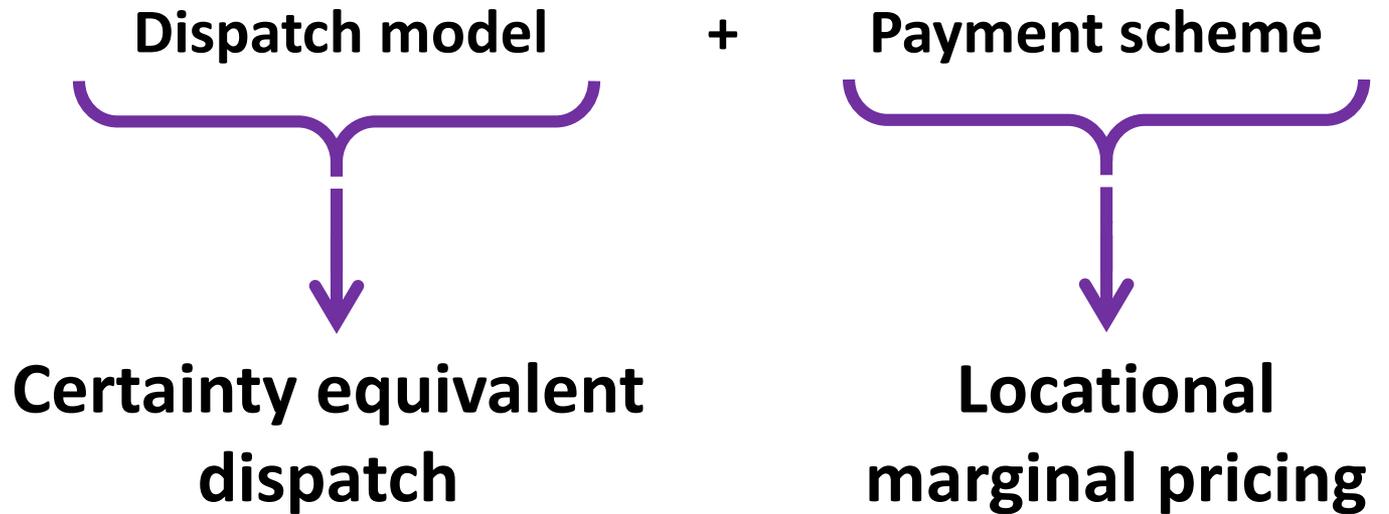
$$x_R^{\square,!} = \min\{!, \mu\},$$

$$\rho^{\square,!} = \square \cdot \mathbb{1}_{\{! < \mu\}}.$$

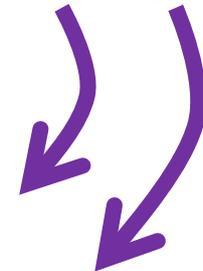
$d > 0$

Uncontrollable but  
predictable demand

# More generally, for a power network...



- Forecast uncertain variables
- Dispatch forward against that forecast
- Re-dispatch to balance attending deviation in real-time



# Problems with the certainty equivalent approach

Forward market clearing process is agnostic to the real-time balancing costs.

Expected total costs:  $d - \mu + \frac{\sqrt{3}}{4} \rho \sigma$



*cost of "waiting"*

*measure of "uncertainty"*

What do forward prices mean?

Does it allow the forward market-participants to hedge their financial risks?



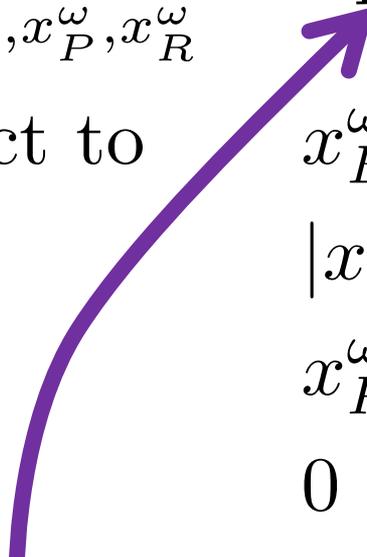
*What prices are deemed meaningful?*

System operator decides the allocation (quantities, prices) through a centralized market clearing process.

*Suppose each generator reflects its true marginal cost to the system operator in its supply offer. Then, she should be willing to produce the prescribed dispatch quantity, when it is paid at the prescribed price. Further, if the generator is a price-taker, it should reflect its true marginal cost to the system operator.*

(Quantities, prices) constitute **an efficient competitive equilibrium.**

# A stochastic economic dispatch model

$$\begin{array}{ll} \text{minimize} & \mathbb{E}_{\mathbf{P}} [1 \cdot x_B^\omega + \rho \cdot x_P^\omega + 0 \cdot x_R^\omega], \\ x_B^\omega, X_B, x_P^\omega, x_R^\omega & \\ \text{subject to} & \left. \begin{array}{l} x_B^\omega + x_P^\omega + x_R^\omega = d, \\ |x_B^\omega - X_B| = 0, \\ x_P^\omega \geq 0, \\ 0 \leq x_R^\omega \leq \bar{x}_R^\omega. \end{array} \right\} \mathbf{P}\text{- a.s.} \end{array}$$


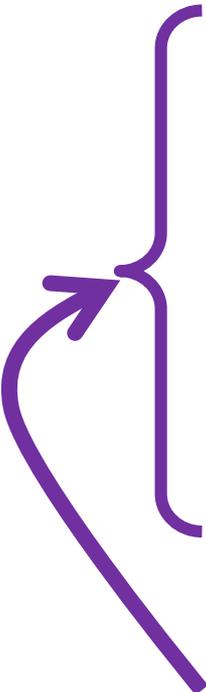
Assume common knowledge.

## *What really constitutes a forward dispatch? Forward prices?*

Introduce extra dispatch quantities  $X_P, X_R$ , and impose demand-supply balance constraint among  $X_B, X_P, X_R, d$ . Pritchard '10.

Other schemes: Galiana '05, Wong '07, Bouffard '08, Morales '09, '12.

# A stochastic economic dispatch model

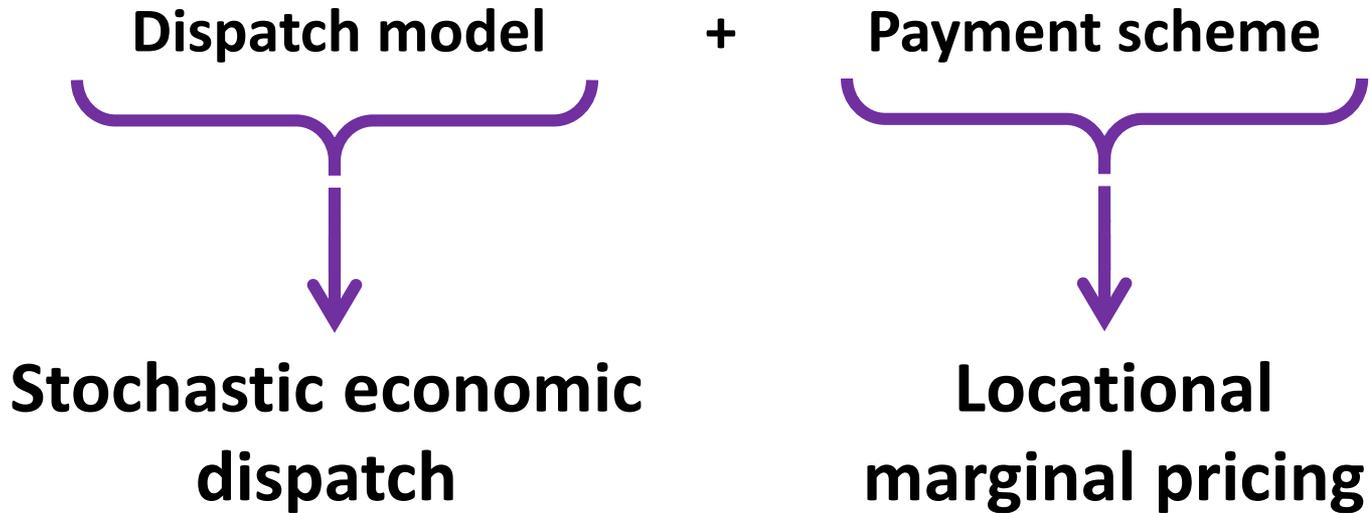
- 
- Enforce forward balance constraint among ***non-physical*** variables, and price such a constraint.
  - Can be ***revenue inadequate*** for certain realizations of available renewable supply.

## *What really constitutes a forward dispatch? Forward prices?*

Introduce extra dispatch quantities  $X_P$ ,  $X_R$ , and impose demand-supply balance constraint among  $X_B$ ,  $X_P$ ,  $X_R$ ,  $d$ . Pritchard '10.

Other schemes: Galiana '05, Wong '07, Bouffard '08, Morales '09, '12.

# Contingent pricing approach



- Defines a dispatch policy

*Defines a pricing policy*

# Contingent pricing for stochastic economic dispatch

$$\underset{x_B^\omega, X_B, x_P^\omega, x_R^\omega}{\text{minimize}} \quad \mathbb{E}_{\mathbf{P}} [1 \cdot x_B^\omega + \rho \cdot x_P^\omega + 0 \cdot x_R^\omega],$$

subject to

$$x_B^\omega + x_P^\omega + x_R^\omega = d,$$

$$|x_B^\omega - X_B| = 0,$$

$$x_P^\omega \geq 0,$$

$$0 \leq x_R^\omega \leq \bar{x}_R^\omega.$$

**P**- a.s.

Market outcome:  $x_B^{*,\omega}, x_P^{*,\omega}, x_R^{*,\omega}, p^{*,\omega}$  for each  $\omega \in \Omega$ .

# Contingent pricing for stochastic economic dispatch

- At the forward stage:
  - The system operator solves the stochastic economic dispatch problem
  - Announces the dispatch policy, and the pricing policy  $x_B^{*,\omega}$ ,  $x_P^{*,\omega}$ ,  $x_R^{*,\omega}$ ,  $p^{*,\omega}$
- In real-time:
  - Enforce the computed dispatch policy
  - Pay according to the pricing policy

# Contingent pricing for stochastic economic dispatch

## *Advantages:*

- Supports an efficient competitive equilibrium
- Revenue adequate over a network in all scenarios

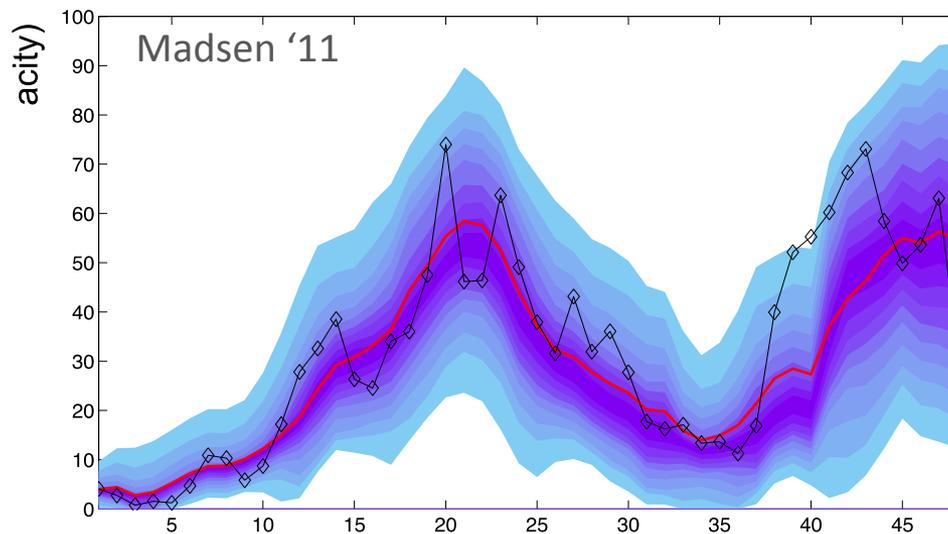
## *Limitations:*

- Communicating a “policy” can be challenging
- Takes away the ability of forward markets to hedge against volatility in real-time markets

“On the design of wholesale electricity markets under uncertainty”,  
S. Bose. Proceedings of the 53<sup>rd</sup> Annual Allerton Conference, 2015.

“Contingent pricing approach to wholesale electricity markets under uncertainty”, S. Bose and E. Bitar. In preparation.

# Do market structures need to change with deepening penetration of variable renewable supply?

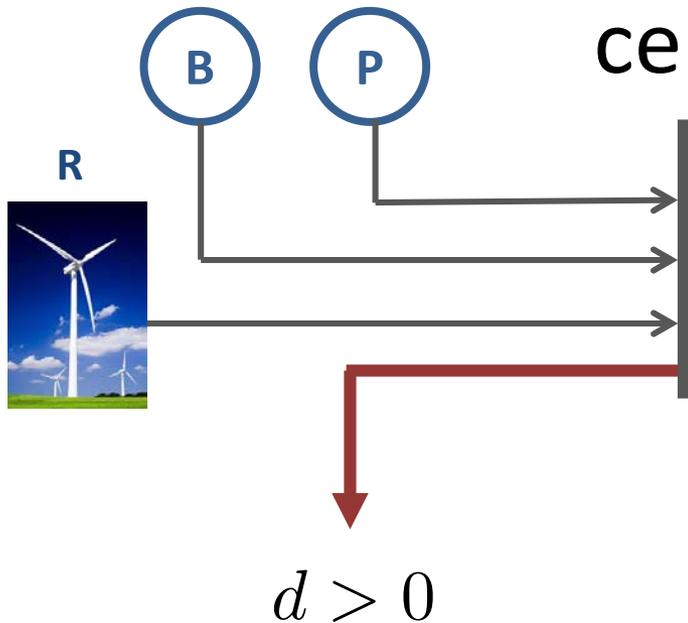


*Uncertain*  
*Intermittent*  
*Non-dispatchable*

**Q1.** *Do we need to change the forward market design?*

**Q2.** *How can we design instruments to mitigate financial risks?*

# Volatility of payments from certainty equivalent approach



Payments to market participants:

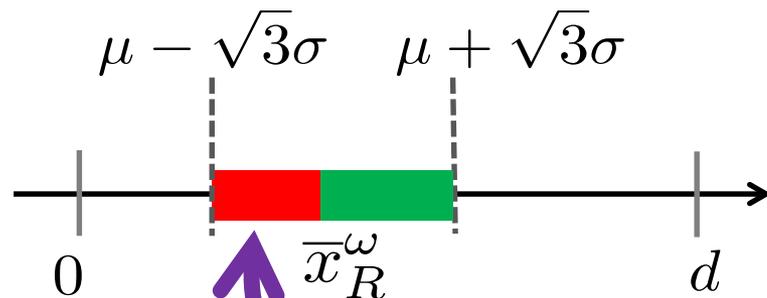
$$\pi_B^\omega = d - \mu,$$

$$\pi_P^\omega = \rho(\mu - \omega)^+,$$

$$\pi_R^\omega = \mu - \rho(\mu - \omega)^+.$$

$\text{var}[\pi^\omega]$  : a measure of volatility

**Q1. Can we reduce this volatility?**

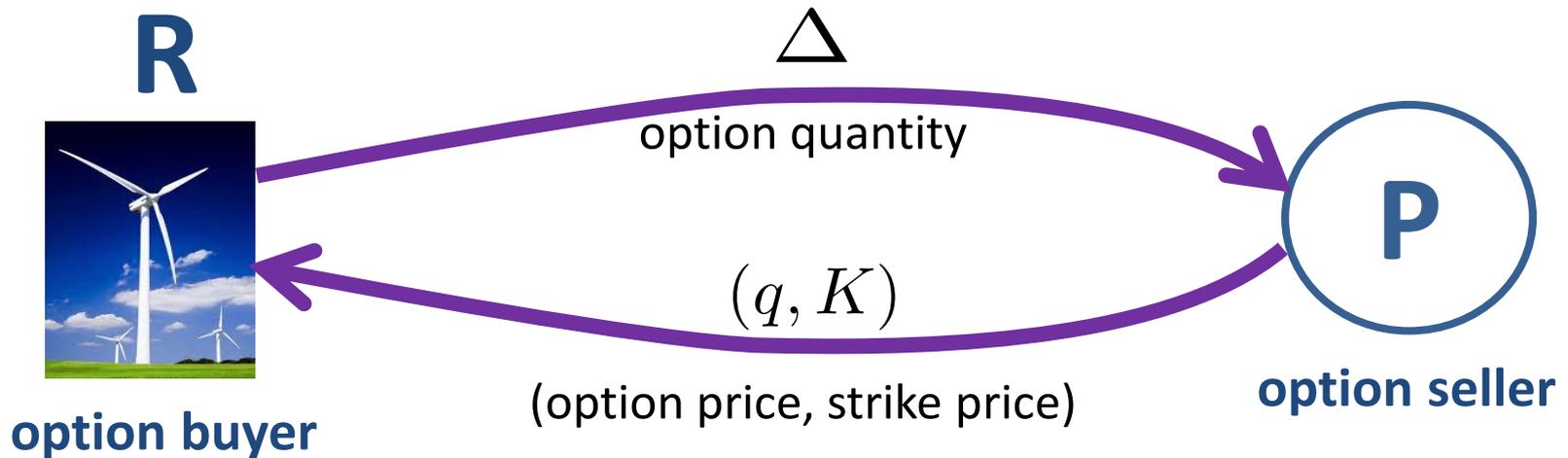


Set over which R gets negative payments  
 $[\mu - \sqrt{3}\sigma, \mu(1 - 1/\rho))$

**Q2. Can we shrink this set?**

partial  
**A** remedy: *bilateral* cash-settled call option

*Assume that players are risk-neutral, and have correct price conjectures.*



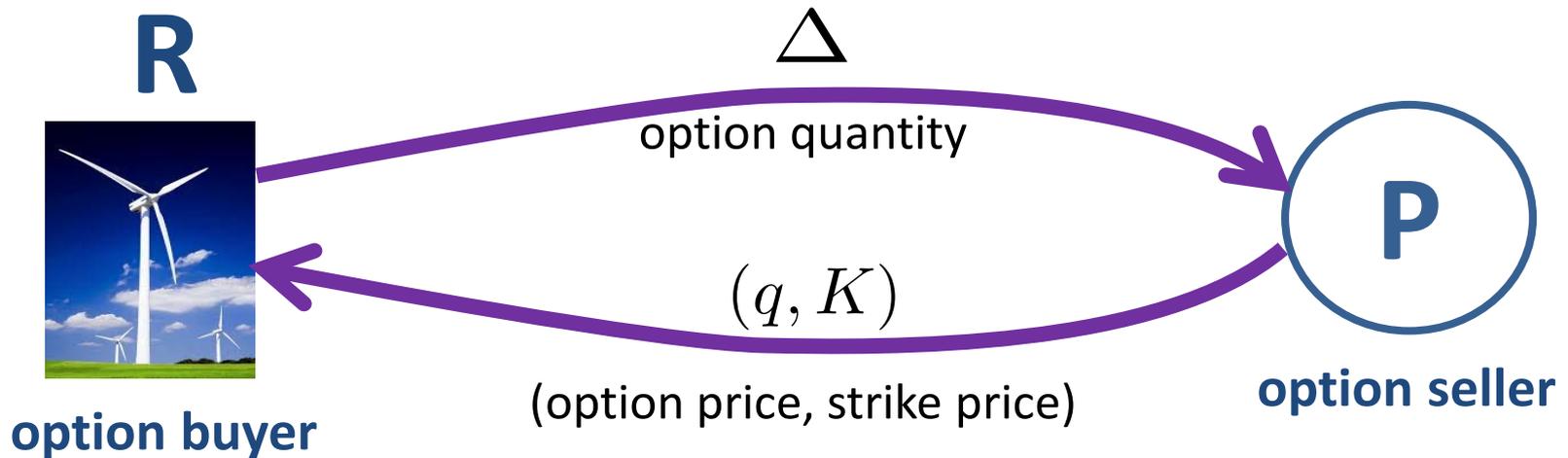
$$\Pi_R^\omega(q, K, \Delta) := \pi_R^\omega - q\Delta + (p^{\omega,*} - K)^+ \Delta,$$

$$\Pi_P^\omega(q, K, \Delta) := \underbrace{\pi_P^\omega}_{\text{Payment from electricity market}} + \underbrace{q\Delta - (p^{\omega,*} - K)^+ \Delta}_{\text{Payment from option trade}}.$$

Payment from  
electricity market

Payment from  
option trade

# Outcomes identified as Stackelberg equilibria



**Definition.**  $(q^*, K^*, \Delta^*) \in \mathbb{R}_+^2 \times \mathbb{R}_+^2 \rightarrow [0, \sqrt{3}\sigma]$  constitutes a Stackelberg equilibrium, if

$$\begin{aligned} \mathbb{E} [\Pi_P^\omega(q^*, K^*, \Delta^*(q^*, K^*))] &\geq \mathbb{E} [\Pi_P^\omega(q, K, \Delta^*(q, K))], \\ \mathbb{E} [\Pi_R^\omega(q, K, \Delta^*(q, K))] &\geq \mathbb{E} [\Pi_R^\omega(q, K, \Delta(q, K))], \end{aligned}$$

for any  $(q, K, \Delta) \in \mathbb{R}_+^2 \times \mathbb{R}_+^2 \rightarrow [0, \sqrt{3}\sigma]$ .

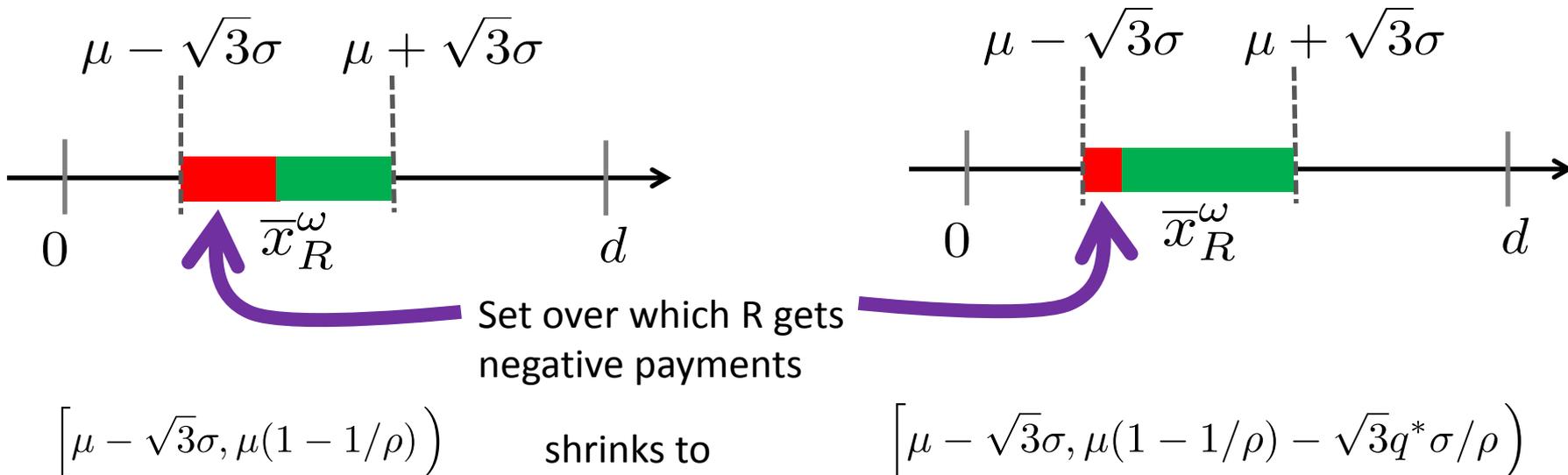
**Proposition.** The nontrivial Stackelberg equilibria are given by

$$\mathcal{N} = \left\{ (q, K, \Delta) \mid (q, K) \in \mathbb{R}_+^2, \Delta : \mathbb{R}_+^2 \rightarrow [0, \sqrt{3}\sigma], 2q + K = \rho \right\}.$$

For  $i \in \{R, P\}$  and any  $(q^*, K^*, \Delta^*(q^*, K^*) = \sqrt{3}\sigma) \in \mathcal{N}$ , we have

$$\text{var} [\Pi_i^\omega(q^*, K^*, \Delta^*(q^*, K^*))] - \text{var} [\pi_i^\omega] = -3K^*\sigma^2/2 < 0.$$

*measure of "uncertainty"*



partial

# A remedy: bilateral cash-settled call option



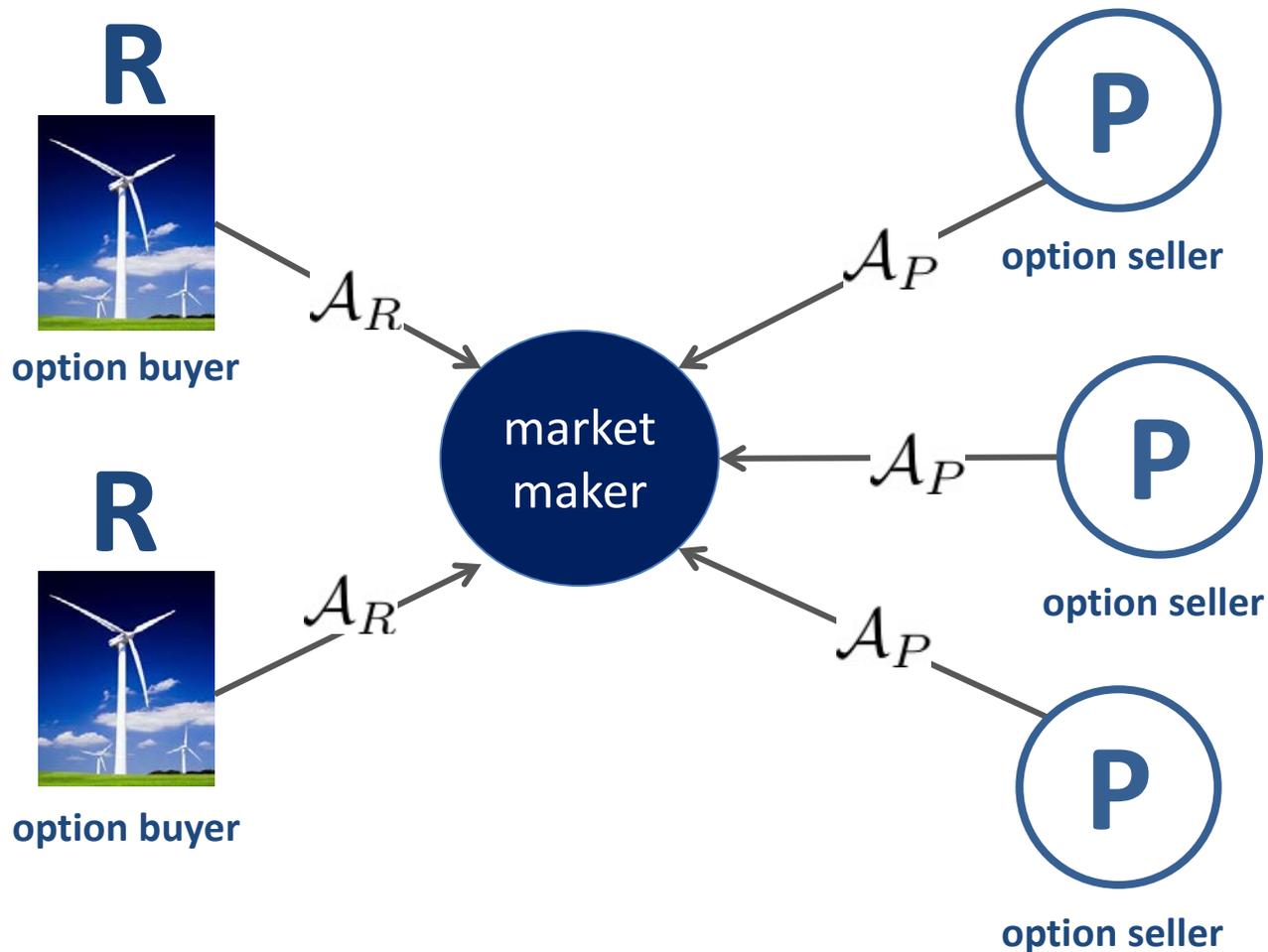
Needs multiple such trades with a collection of dispatchable and renewable power producers



**Key idea:** introduce an intermediary we call the market maker

“Cash-settled options for wholesale electricity markets”, K. Alshehri, S. Bose, and T. Başar. To appear in the proceedings of the 20th IFAC World Congress, 2017.

# Market maker buys from option sellers and sells them to option buyers



$\mathcal{A}$  defines the set of acceptable trades  $(q, K, \Delta)$  for each participant.

# Market maker buys from option sellers and sells them to option buyers

*...a centralized option trading mechanism*

*...generalizes the bilateral trade case*

Option market clearing via stochastic optimization:

maximize  $E_X[MS^*]$ ,  
 subject to  $\Delta_P = \Delta_R$ ,  
 $(q_P, K_P, \Delta_P) \in A_P, (q_R, K_R, \Delta_R) \in A_R$ ,  
 $\delta_P^i \in [0, \Delta_P]$ ,  
 $\delta_P^i = \Delta_R \mathbb{1}_{\{p^i \geq K_R\}}$   
 for each  $P \in \mathcal{G}, R \in \mathcal{R}, i \in \mathcal{I}$

*Market maker may have a different objective.*

*Can encode risk aversion in the set of acceptable trades.*

$$MS^\omega := \sum_{R \in \mathcal{R}} [q_R \Delta_R - (p^{\omega,*} - K_R)^+ \Delta_R] - \sum_{P \in \mathcal{P}} [q_P \Delta_P - (p^{\omega,*} - K_P)^+ \delta_P^\omega].$$



**Subhonmesh Bose**

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**Thank you!**