DER Coordination at Large Scales:
from Mean Field Games to Social Welfare Optimization

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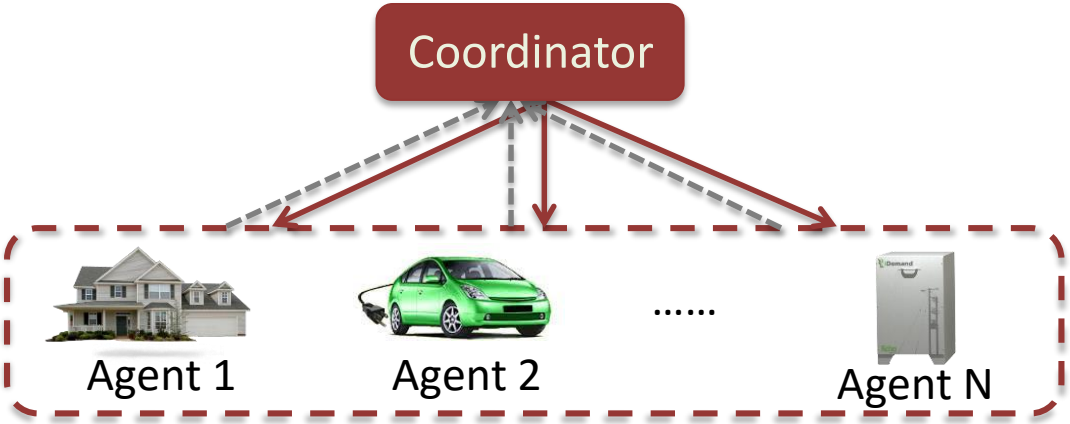
Outline

- **Part I: Mean Field Game for Large Scale DER Coordination**
  - Background
  - Main results: Mean field Game vs. social welfare optimization
  - Numerical Example
  - Extension to Nonconvex Case
- **Part II: Cross-Layer Design of DER Coordination via Aggregate Load Modeling**

Background

- DERs in future power grid: opportunities + challenges



- Users and stakeholders of DERs are self-interested
- Coordination strategies must respect users' preferences and possibly strategic behaviors
  - Opt-out from the program
  - Game the system

Background

$$\max_{u_1} U_1(u_1, u_2, \dots, u_N)$$
$$u_1 \in U_1$$
Agent 1

$$\max_{u_i} U_i(u_1, u_2, \dots, u_N)$$
$$u_i \in U_i$$
Agent i

$$\max_{u_N} U_N(u_1, u_2, \dots, u_N)$$
$$u_N \in U_N$$
Agent N

- Classical game theory has been widely used in power systems
- **Emerging challenges:**
  - Potentially large number of DERs
  - DERs typically have nontrivial dynamics

From Game to Mean-Field Game

$$\max_{u_1} U_i(u_1, \boldsymbol{\Sigma_j u_j}) \quad u_1 \in U_1$$ Agent 1

$$\max_{u_i} U_i(u_i, \boldsymbol{\Sigma_j u_j}) \quad u_i \in U_i$$ Agent i

$$\max_{u_N} U_i(u_N, \boldsymbol{\Sigma_j u_j}) \quad u_N \in U_N$$ Agent N

- Often times, "weak" coupling depends only on aggregate effect
- Mean-field game theory can simplify the analysis of the game

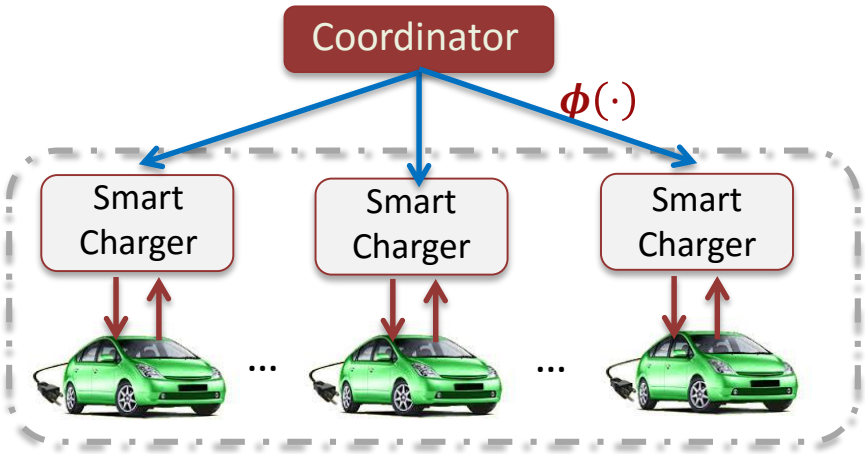
$(u_1^*, \dots, u_N^*)$ is *Nash* equilibrium if $U_i(u_i^*, u_{-i}^*) \geq U_i(u_i, u_{-i}^*), \forall\, u_i \in U_i$

$(u_1^*, \dots, u_N^*)$ is **ϵ-Nash** equilibrium if $U_i(u_i^*, u_{-i}^*) \geq U_i(u_i, u_{-i}^*) - \boldsymbol{\epsilon}, \forall\, u_i \in U_i$

Typically require: $\epsilon \to 0$ as $N \to \infty$

- Computation/analysis of ϵ-Nash (Mean-field equilibrium) is easier but not easy.

Simple Example (PEV Charging)

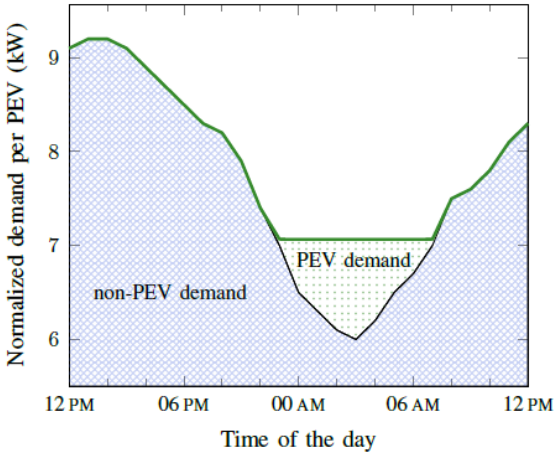


$$\min_{(u_{i,1},\dots,u_{i,T})} \sum_t^T p_t(\bar{u}) \cdot u_{i,t} + \eta \left(u_{i,t} - \bar{u}_{i,t}\right)^2$$

Subject to: $x_{i,t+1} = x_{i,t} + u_{i,t},\ x_{i,T} = \gamma_i,$

$$p_t(\bar{u}) = \alpha \bar{u}_{i,t} + d_t,\quad 0 \le u_{i,t}, \le u_i^{\max}$$

- Cooperative PEV charging (Ma12), (Grammatico15)
  - $\boldsymbol{u_i} = \left(u_{i,1}, \dots, u_{i,T}\right)$ is energy profile
  - Objective: energy cost + small penalty for deviation from mean
  - Electricity price depends on total demand: $\left(\sum_i u_{i,t} + d_t\right)$
  - The coupling pricing induces a game among all the PEVs
  - Existing results: characterization and computation of the mean field equilibrium

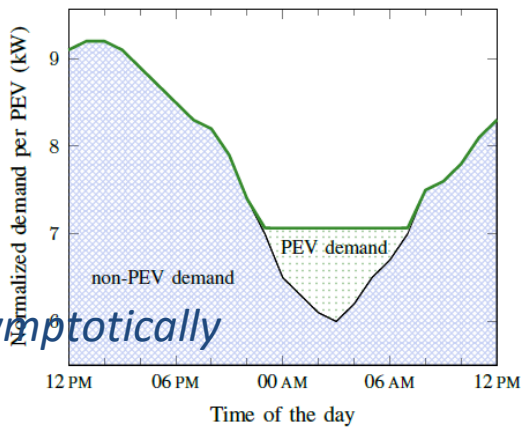


Other Examples in DER Coordination:

- **Thermostatically controlled loads:**
  - (Grammatico 15), (Bauso 14), (Bagagiolo & Bauso 14), (Li, Zhang, Lian and Kalsi 16), (Chen & Meyn 17)
- **Plug-in Electric Vehicles**:
  - (Ma, Callaway & Hiskens 12), (Couillet, Perlaza, Tembine & Debbah 12), (Parise, Colombino, Grammatico & Lygeros 14), (Paccagnan, Kamgarpour & Lygeros 16) and (Deori, Margellos, & Prandini 16)
- **Pool Pump**:
  - (Meyn, Barooah, Bušić, Chen, & Ehren 15) and (Chen, Bušić, & Meyn 14)
- **Others:** (Zhu & Basar 11), (Kamgarpour &Tembine 13)
- In general, no guarantee for successfully computing the mean-field equilibrium

Limitations of Existing Literature

- **Computation:**
  - Existing works mostly focus on deriving mean-field equations (based on Stochastic Differential Equation agent models)
  - Mean-field equilibrium needs to solve: HJB + Fokker Planck
  - Efficient algorithm is not available except for very special cases (Bardi 12, Grammatico, et al 16, Achdou and Porretta 16)



- **System level performance (Efficiency):**
  - Cooperative agents (Huang 2012):
    - Modify agent utilities and achieve social maximum *asymptotically*
  - Non-cooperative agents (mostly negative results):
    - Mean field equilibrium (MFE) is inefficient (Balandat and Tomlin, 2013)
    - Efficiency loss has been studied (Huang, 2007), (Yin, et al, 2013)

Challenges and Our Contribution

- **Key questions** to enable applications on DER coordination:
  - How to efficiently compute a mean-field equilibrium?
  - Is it good or bad?
  - Given system-level objective, how to induce a good mean-field equilibrium?

Contribution: **mean field game** $\longleftrightarrow$ **modified social welfare optimization**

- **Existence, uniqueness, computation** of MFE through optimization
- **Efficiency Evaluation**: MF equilibrium coincides with social optimizer

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A Class of Mean-Field Games in Vector Space

Mean Field Game

$$\max_{\{\boldsymbol{u_i}\}} \mathbb{E}(V_i(x_i, u_i) - \phi(\bar{x}) \cdot x_i + G(\bar{x}))$$

Agent i $\begin{cases} x_i = f_i(u_i, w_i) \\ \mathbb{E}(x_i) \in X_i, \quad u_i \in U_i \end{cases}$

Grammatico, Prise, Columbino, & Lygeros, TAC 16

$$\min_{\{u_{i,t}\}_t} \sum_{t=1}^{K} ||x_{i,t} - \bar{x}_t||^2 + \sum_{t=1}^{K} \|u_{i,t}\|^2$$

$$\begin{cases} x_{i,t+1} = A_i x_{i,t} + B_i u_{i,t} \\ x_{i,t} \in X_{i,t}, u_{i,t} \in U_{i,t} \end{cases}$$

$$V(x_i, u_i) = \|x_i\| + \|u_i\|$$

$$\phi(\bar{x}) = 2\bar{x}, \qquad G(\bar{x}) = \|\bar{x}\|^2$$

- $\bar{x}$ is the mean field: $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$.
- Assumptions:
  - X_i is Hilbert space
  - $\forall u \in U_i \Rightarrow \mathbb{E}(x_i) \in X_i, \mathbb{E}(\|x_i\|^2) \leq C$, where U_i subset of arbitrary vector space
  - w_i is a random element (measurable mapping)
  - Mean field term $\phi(\cdot)$ is Lipschitz continuous
  - $G(\cdot)$ is Frechet differentiable
- Includes both deterministic (Grammatico, et al, 2016) and stochastic (Huang, et al, 2007) as special cases.

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Huang, Caines & Malhame TAC 07

$$\min_{\{u_{i,t}\}_t} \mathbb{E} \int_0^\infty e^{-\rho t} \left[\left(x_{i,t} - \bar{x}_t\right)^2 + r\left(u_{i,t}\right)^2 \right] dt$$

$$dx_{i,t} = (A_i x_{i,t} + B_i u_{i,t})dt + \sigma_i dw_{i,t}$$

Inner product $\quad h \cdot q = \int_0^\infty e^{-\rho t} h_t q_t dt$

$$\min_{\boldsymbol{u_i}} \mathbb{E}\|x_i\|^2 + \|\bar{x}\|^2 - 2x_i \cdot \bar{x} + r\|u_i\|^2$$

- $\bar{x}$ is the mean field: $\bar{x} = \frac{1}{N}\sum_{i=1}^N x_i$.
- Assumptions:
  - X_i is Hilbert space
  - $\forall u \in U_i \Rightarrow \mathbb{E}(x_i) \in X_i, \mathbb{E}(\left|\left|x_i\right|\right|^2) \leq C$, where U_i can be arbitrary vector space
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Mean Field Equilibrium

Mean Field Game

$$\max_{\{u_i\}} \mathbb{E}(V_i(x_i, u_i) - \phi(\bar{x}) \cdot x_i)$$

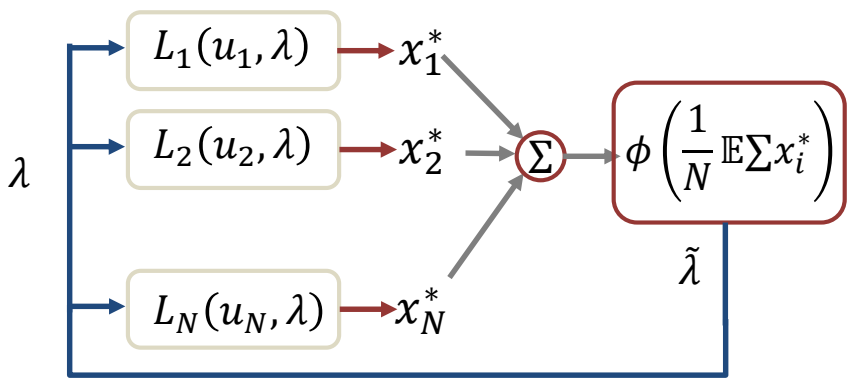
Agent i $\begin{cases} x_i = f_i(u_i, w_i) \\ \mathbb{E}x_i \in X_i, \quad u_i \in U_i \end{cases}$

$(u_1^*, \dots, u_N^*)$ is ϵ-Nash equilibrium if

$$U_i(u_i^*, u_{-i}^*) \geq U_i(u_i, u_{-i}^*) + \epsilon, \forall\, u_i \in \bar{U}_i$$

$$L_i(u_i, \lambda) = \mathbb{E}V_i(x_i, u_i) - \lambda \cdot \mathbb{E}x_i$$

$$\mu_i^*(\lambda) = \arg\max_{u_i \in U_i} L_i(u_i, \lambda)$$



Mean Field Fixed Point Equations

$$u_i^* = \arg\max_{u_i \in U_i} L_i(u_i, \boldsymbol{\lambda})$$

$$x_i^* = f_i(u_i^*, w_i) \quad \boldsymbol{\lambda} = \phi(\tfrac{1}{N}\,\mathbb{E}\Sigma x_i^*)$$

Theorem 1: The solution, $(u_1^*, \dots, u_N^*)$, to the mean-field equations is an ϵ_N-Nash equilibrium of the mean field game, and $\lim_{N\to\infty} \epsilon_N = 0$.

Unified result, more general than the literature

Mean Field Game to Social Welfare Optimization

Mean Field Game

$$\max_{\{u_i\}} \mathbb{E}(V_i(x_i, u_i) - \phi(\bar{x}) \cdot x_i)$$

Agent i $\begin{cases} x_i = f_i(u_i, w_i) \\ \mathbb{E}x_i \in X_i, \quad u_i \in U_i \end{cases}$

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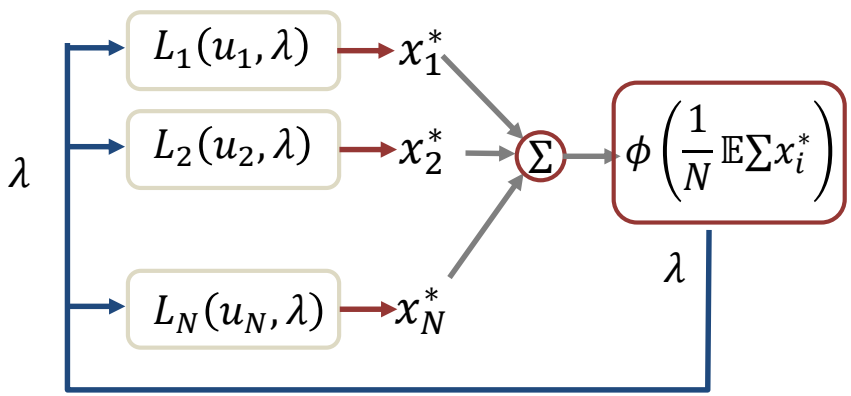

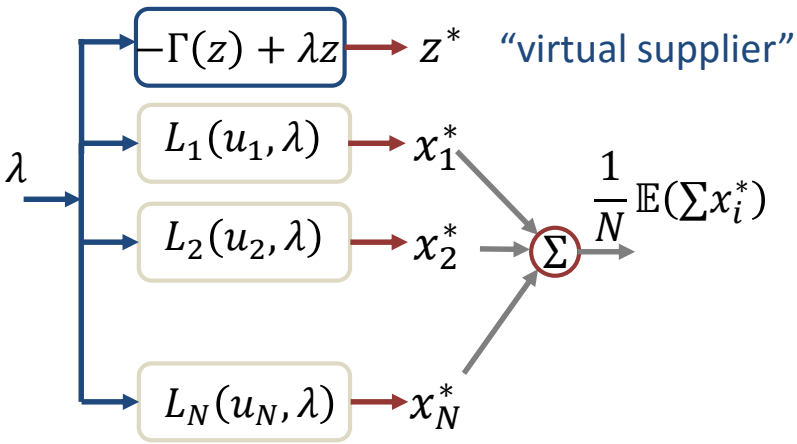

- View mean-field effect λ as "price"
- Becomes standard social welfare problem: optimal price is "marginal" production cost

$$z^* = \frac{1}{N}\mathbb{E}(\sum x_i^*) \qquad \lambda = \Gamma'(z) = \phi\left(\frac{1}{N}\mathbb{E}\sum x_i^*\right) \quad \Rightarrow \quad \phi(\cdot) = \frac{1}{N}\Gamma'(\cdot)$$

Main Results

Mean Field Game

$$\max_{\{u_i\}} \mathbb{E}(V_i(x_i, u_i) - \phi(\bar{x}) \cdot x_i)$$

Agent i $\begin{cases} x_i = f_i(u_i, w_i) \\ \mathbb{E}x_i \in X_i, \quad u_i \in U_i \end{cases}$

$\phi(\cdot) = \frac{1}{N}\Gamma'(\cdot)$

Social Welfare Optimization

$$\max_{\{u_1, \ldots u_N, z\}} \mathbb{E}\left(\sum_{i=1}^{N} V_i(x_i, u_i) - \Gamma(z)\right)$$

$$x_i = f_i(u_i, w_i)$$

$$\mathbb{E}x_i \in X_i, u_i \in U_i, z = \frac{1}{N}\mathbb{E}(\sum x_i)$$

Theorem 2: If the above social optimization problem is strictly convex, its solution coincides with the solution to mean-field equations.

Main Results

Mean Field Game

$$\max_{\{u_i\}} \mathbb{E}(V_i(x_i, u_i) - \phi(\bar{x}) \cdot x_i)$$

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$$\mathbb{E}x_i \in X_i, u_i \in U_i, z = \frac{1}{N}\mathbb{E}(\sum x_i)$$

- **Significance:**
  - Connection between mean field equilibrium with social optimal solution
    - **Mean field equilibrium is socially optimal**
    - **Social optimal solution is ϵ-Nash**
- Unified framework to study an important class of mean-field games
  - Huang 03, Couillet 12, Bauso 14, Huang 10, Ma 12, Kamgarpour 13, Tembine 13, Grammatico 15, Grammatico 16,
- Finding mean-field equilibrium is a convex optimization problem

Mean-Field Equilibrium Computation

Mean Field Game

$$\max_{\{u_i\}} \mathbb{E}(V_i(x_i, u_i) - \phi(\bar{x}) \cdot x_i)$$

Agent i $\begin{cases} x_i = f_i(u_i, w_i) \\ \mathbb{E}x_i \in X_i, \quad u_i \in U_i \end{cases}$

$$\begin{pmatrix} u_1^{(k)} \\ \vdots \\ u_N^{(k)} \end{pmatrix} \longrightarrow \sum \alpha_i x_i^{(k)} \xrightarrow{\Phi(\lambda^{(k)}, \sum \alpha_i x_i^{(k)})} \lambda^{(k+1)}$$

"best response"

- Existing literature: either no computation or simple iterations
  - (Grammatico, Gentile, Parise & John Lygeros 15), (Bauso 14), (Bagagiolo & Bauso 14), (Ma, Callaway & Hiskens 12), (Couillet, Perlaza, Tembine & Debbah 12), (Parise, Colombino, Grammatico & Lygeros 14), (Paccagnan, Kamgarpour & Lygeros 16) and (Deori, Margellos, & Prandini 16)
- In general, no convergence guarantees
- (Grammatico, Prise, Columbino, & Lygeros, TAC 16): Various conditions on convergence for special linear quadratic cases

Mean-Field Equilibrium Computation

Mean Field Game

$$\max_{\{\boldsymbol{u_i}\}} \mathbb{E}(V_i(x_i, u_i) - \phi(\bar{x}) \cdot x_i)$$

Agent i $\begin{cases} x_i = f_i(u_i, w_i) \\ \mathbb{E}x_i \in X_i, \quad u_i \in U_i \end{cases}$

$\phi(\cdot) = \frac{1}{N}\Gamma'(\cdot)$

Social Welfare Optimization

$$\max_{\{\boldsymbol{u_1}, \ldots \boldsymbol{u_N}, \boldsymbol{z}\}} \mathbb{E}\left(\sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} V_i(x_i, u_i) - \Gamma(z)\right)$$

$$x_i = f_i(u_i, w_i)$$

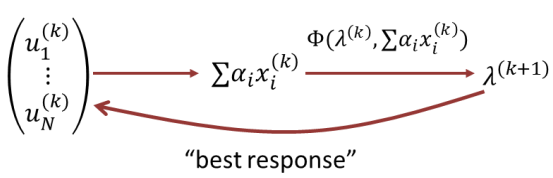
$$\mathbb{E}x_i \in X_i, u_i \in U_i, z = \frac{1}{N}\mathbb{E}(\sum x_i)$$

Dual Form

$$\min_{\lambda} \max_{\{u_1, \ldots u_N, z\}} \mathbb{E}[\sum(V_i(x_i, u_i) - \lambda \cdot x_i)] - \Gamma(z) + \lambda \cdot Nz$$

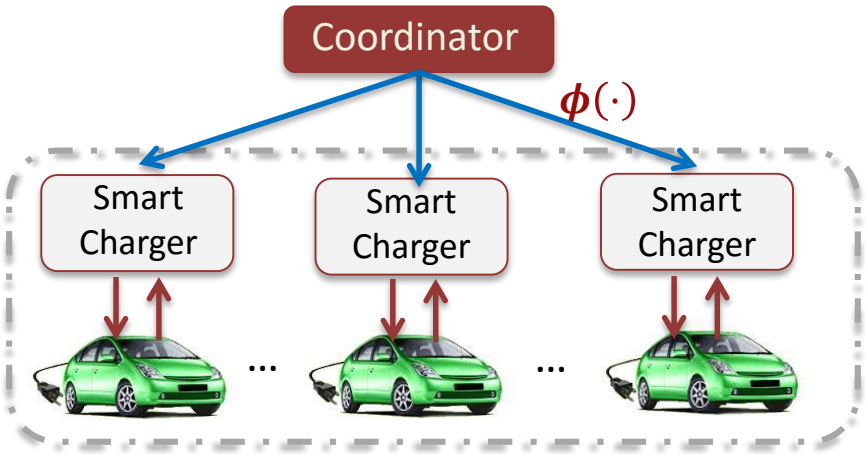
$$\begin{cases} x_i = f_i(u_i, w_i), & \forall i = 1, \ldots, N \\ \mathbb{E}x_i \in X_i, u_i \in U_i, & \forall i = 1, \ldots, N \end{cases}$$

- Existing iterative equilibrium algorithms are essentially prime dual



- Extensive literature on convex optimization can be used to facilitate equilibrium computation (e.g. ADMM)

PEV Charging Example (Revisited)



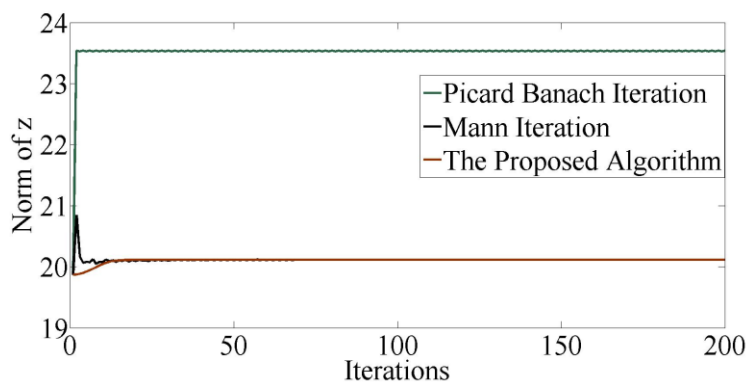
$$\min_{(u_{i,1},\ldots,u_{i,T})} 2a(\sum \boldsymbol{u_i} + \boldsymbol{d})^T \boldsymbol{u_i} + \eta \, \|\boldsymbol{u_i} - \bar{\boldsymbol{u}}\|^2$$

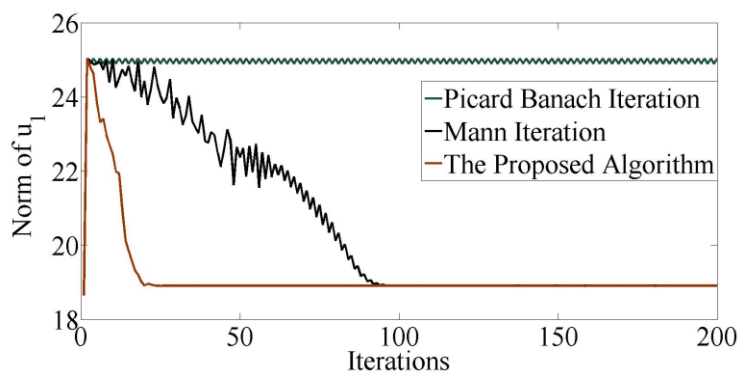
subject to: $\boldsymbol{u_i} \in U_i = \{u_{i,t} \geq 0, \sum_t u_{i,t} = \gamma^i\}$

$$\min_{(u_1,\ldots,u_N,z)} \eta \sum_i \|\boldsymbol{u^i} - \boldsymbol{z}\|^2 + \Gamma(Nz)$$

subject to: $\sum_t u_i(t) = \gamma^i \quad \frac{1}{N}\sum_i u_i = z$

- Our algorithm: construct the social welfare optimization, and solve it via ADMM
- Benchmark algorithm (Grammatico 16): given z, derive $\boldsymbol{u_i}$, then update z





Extensions to Nonconvex Case

Mean Field Game

$$\max_{\{u_i\}} \mathbb{E}(V_i(x_i,u_i) - \phi(\bar{x}) \cdot x_i)$$

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$\phi(\cdot) = \frac{1}{N}\Gamma'(\cdot)$

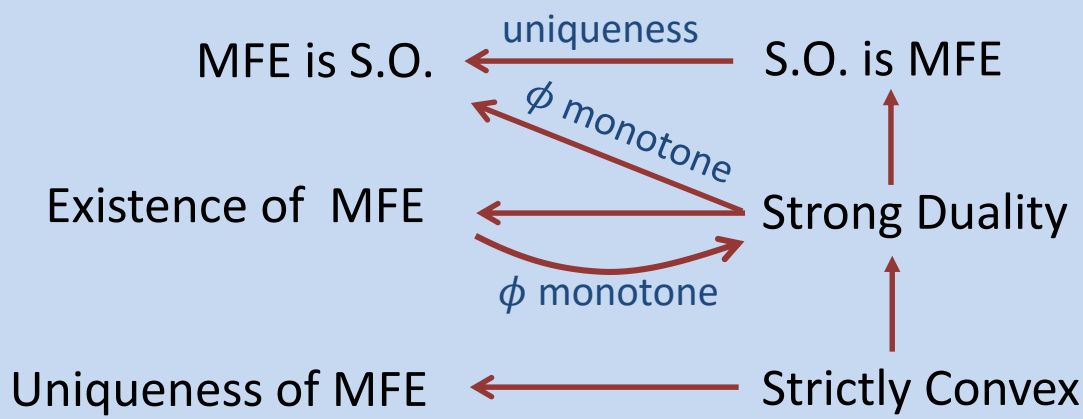
Social Welfare Optimization

$$\max_{\{u_1,\dots u_N, z\}} \mathbb{E}\left(\sum_{i=1}^{N} V_i(x_i,u_i) + \Gamma(z)\right)$$

$$x_i = f_i(u_i, w_i)$$

$$\mathbb{E}x_i \in X_i, u_i \in U_i, z = \frac{1}{N}\mathbb{E}(\sum x_i)$$

Theorem 3 (Complete Connections):



- Recover and extend results in the literature
- Novel perspective to study mean field games;

ϕ monotone: $\langle \phi(x) - \phi(y), x - y \rangle \geq 0$

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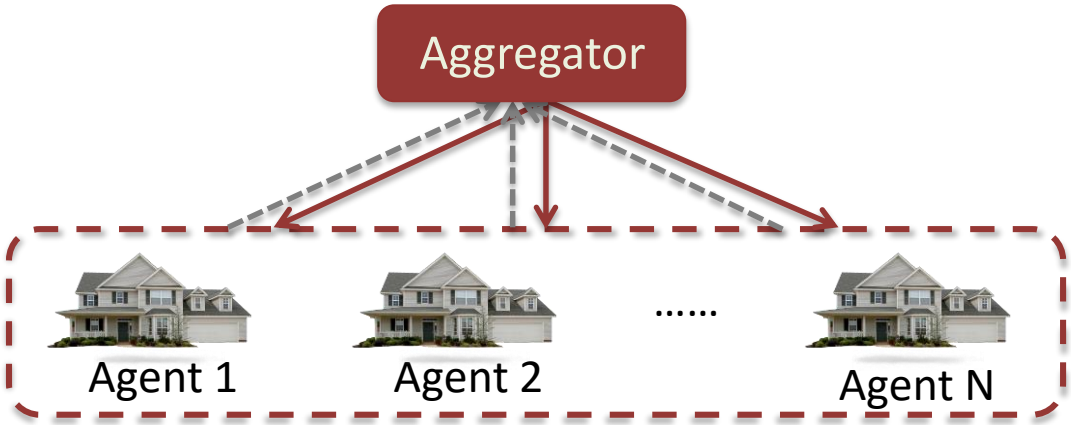
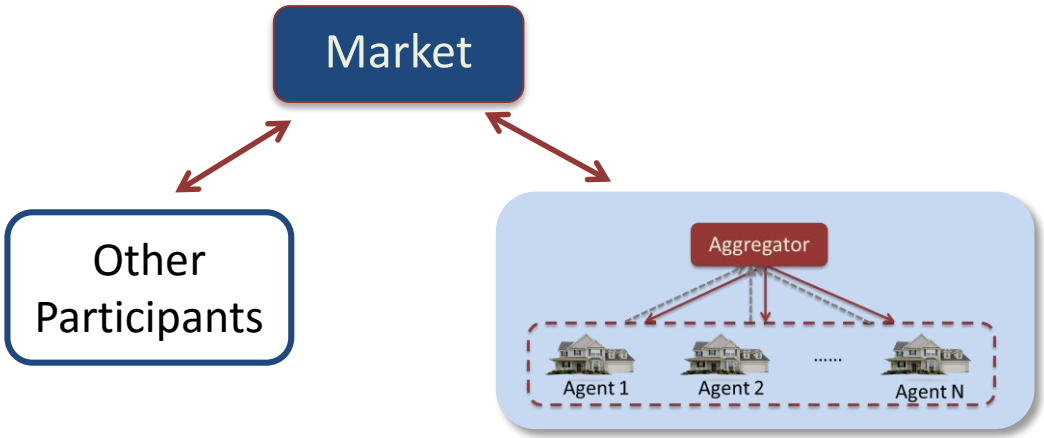
Towards Cross-Layer Design



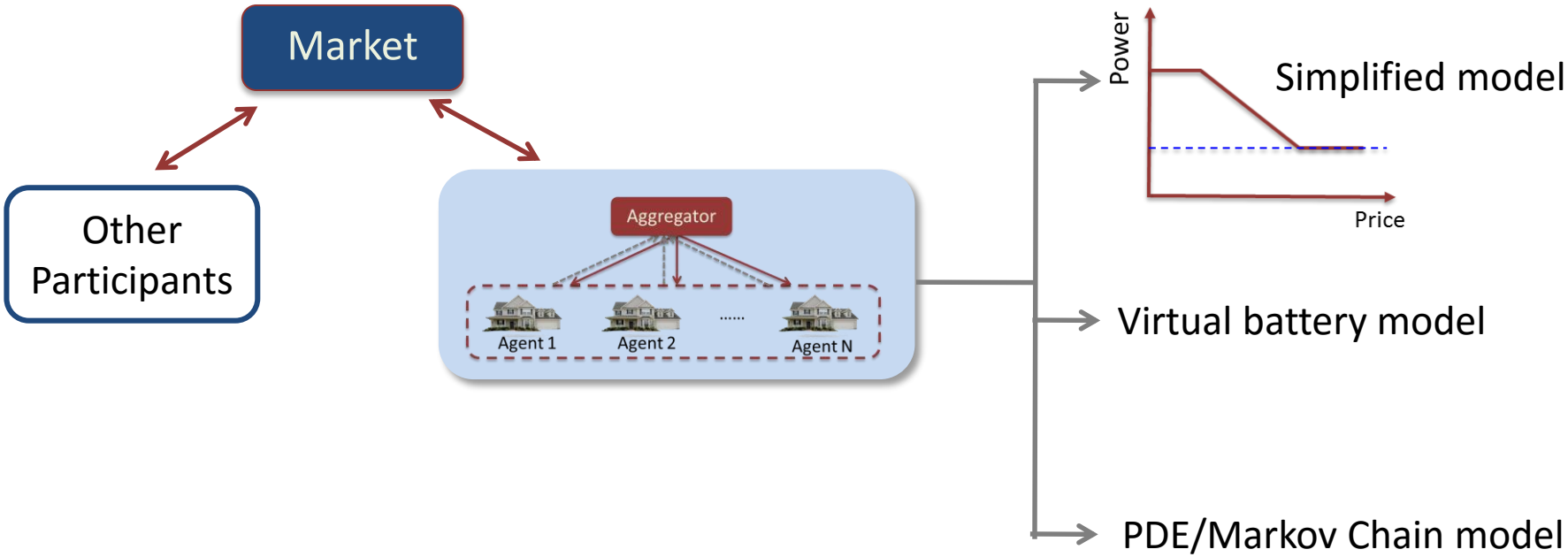
- Coordination of aggregation of DERs can reshape total power profile
  - Market-based (Transactive)
  - Direct load control

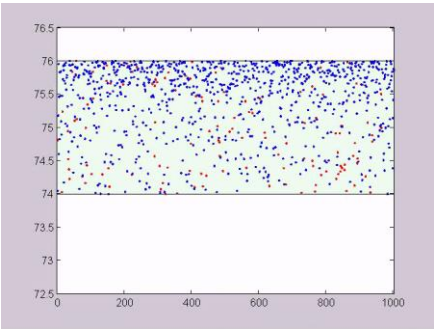
Towards Cross-Layer Design



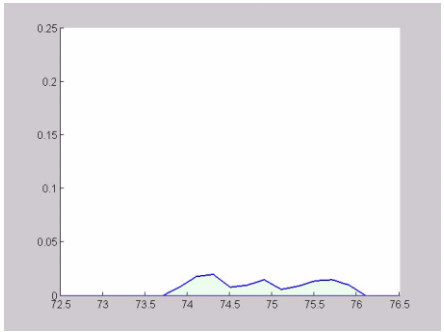
- Coordination of aggregation of DERs can reshape total power profile
  - Market-based (Transactive)
  - Direct load control
- Need to interact with higher layer (e.g. whole sale market)
- **Key:** need a good aggregate asset model
  - Understand how much we can modify the power profile

Towards Cross-Layer Design

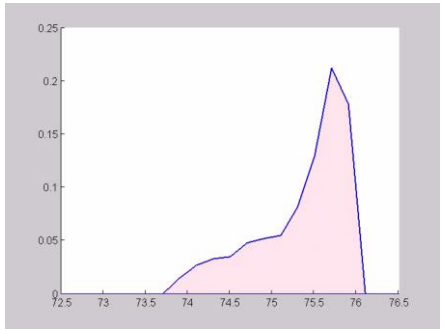




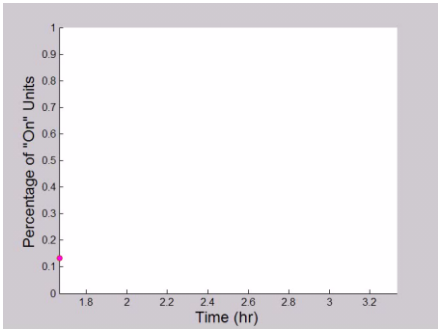
Air temperature evolution



"ON" Distribution

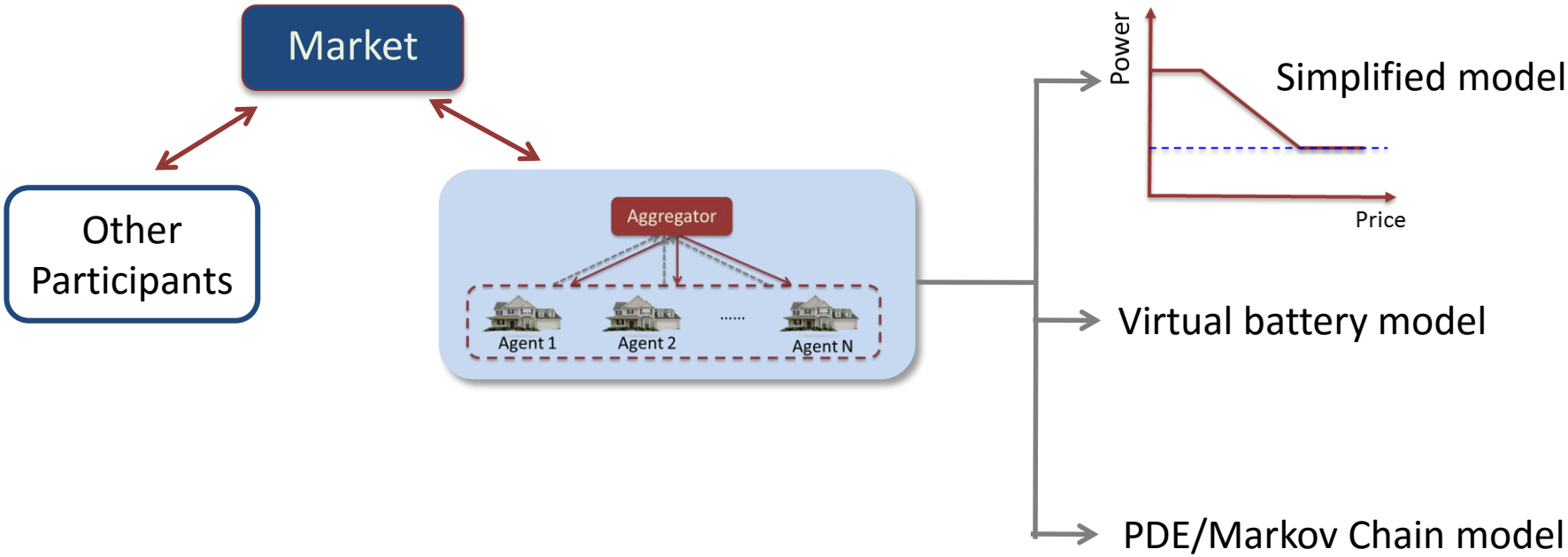


"OFF" Distribution



Percentage of "ON" Units

Towards Cross-Layer Design



- We developed a geometric approach for aggregate flexibility modeling
  - Include virtual battery modeling as special cases
  - Automate and optimize virtual battery modeling process
  - Improve modeling performance

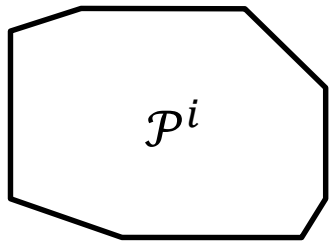
Geometric Approach for Aggregate Flexibility

- Individual asset dynamic model

$$x^i(k) = \alpha^i x^i(k-1) + b^i u^i(k),$$
$$u^i \in [-u^i_-, u^i_+], \quad x^i \in [-x^i_-, x^i_+]$$

- Individual power flexibility over $[0, m]$

$$\mathcal{P}^i := \left\{ u^i \in \mathbb{R}^m \middle| \begin{array}{c} u^i(t) \in [-u^i_-(t), u^i_+(t)], \\ u^i(t), x^i(t) \text{ satisfy (1)}, \\ x^i(t) \in [-x^i_-(t), x^i_+(t)] \end{array} \right\}$$



- Aggregate power flexibility over $[0, m]$

$$\mathcal{P} = \{u \in \mathbb{R}^m : \; u = \Sigma_i^N u^i, \quad u^i \in \mathcal{P}^i \}$$

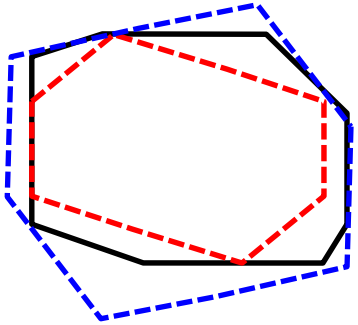
This is Minkowski sum: $\mathcal{P} := \biguplus_i \mathcal{P}^i$

Geometric Approach for Aggregate Flexibility

- Calculation of the Minkowski sum of $\mathcal{P}^i$ is NP hard

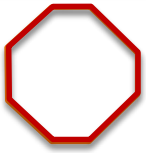
- Look for inner (sufficient) and outer (necessary) approximation of $\mathcal{P}$

$$\mathcal{P}_s \subset \mathcal{P} \subset \mathcal{P}_n.$$



- **Key Fact**: Minkowski sum of homothets reduces to ordinary sum:

$$(\beta_1 \mathcal{Q} + t_1) \oplus (\beta_2 \mathcal{Q} + t_2) = (\beta_1 + \beta_2)\mathcal{Q} + (t_1 + t_2).$$



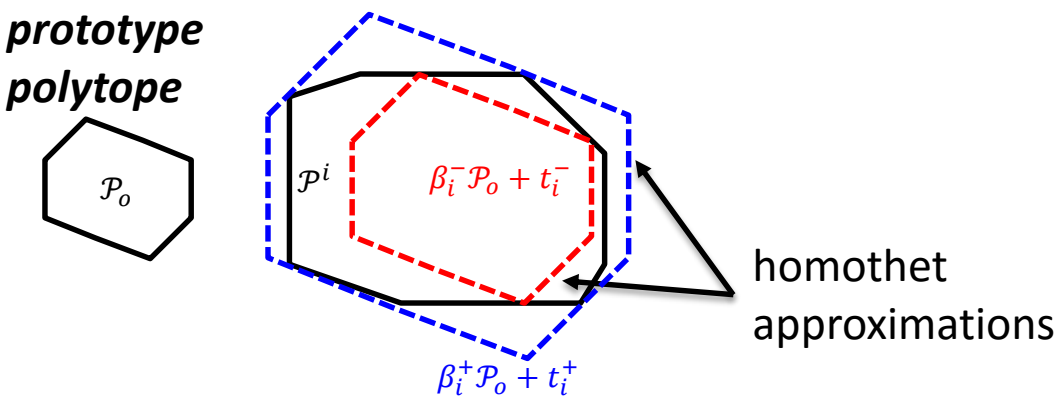
$\beta\mathcal{Q} + t$

$\mathcal{Q}$

Homothet:
dilation and translation of an object,
$\forall \beta > 0, \beta\mathcal{Q} + t := \{U : U = \beta\xi + t, \forall \xi \in \mathcal{Q}\}$

Geometric Modeling Procedure

- **Modeling Procedure:**
  1. Choose a prototype polytope $\mathcal{P}_o$
  2. Approximate each $\mathcal{P}^i$ by the homothet of $\mathcal{P}_o$
  3. Calculate the Minkowski sum of these homothetic approximations



Minimum Outer Appr. (MOA)

$$\begin{aligned} \text{minimize} \quad & \beta \\ \text{subject to:} \quad & \beta \mathcal{P}_o + t \supset \mathcal{P}^i, \\ & \beta > 0, \end{aligned}$$

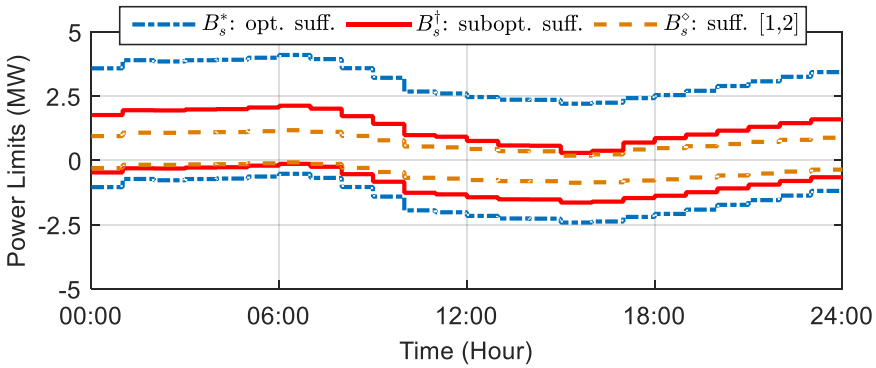
Maximum Inner Appr. (MIA)

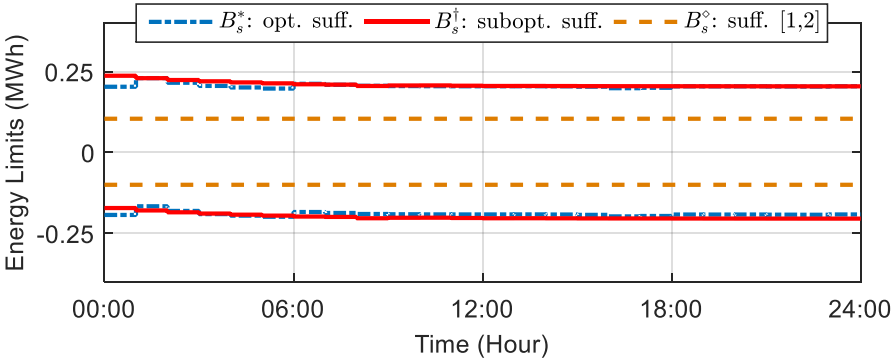
$$\begin{aligned} \text{maximize} \quad & \beta \\ \text{subject to:} \quad & \beta \mathcal{P}_o + t \subset \mathcal{P}^i, \\ & \beta > 0, \end{aligned}$$

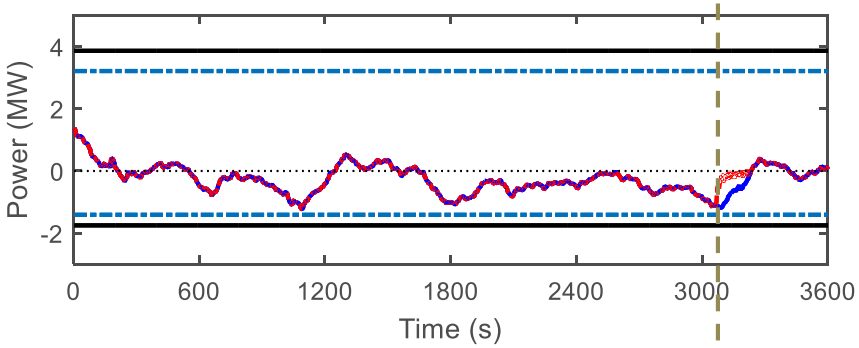
- If $\mathcal{P}_o$ is chosen from a virtual battery model, the resulting approximation has a (virtual) battery interpretation
- Many existing virtual battery modeling methods are special cases

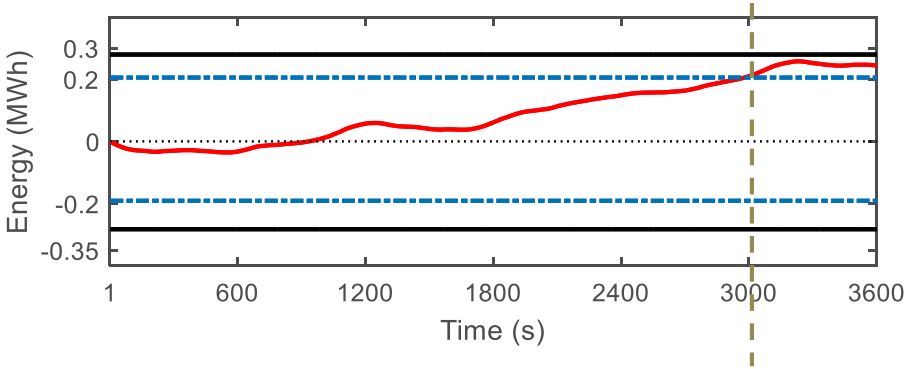
Numerical Validations

- 1000 residential HVACs
- Priority-stack-based tracking control of hybrid TCL models
- Regulation capacity: estimated using sufficient virtual battery model









[1] He Hao, Borhan M. Sanandaji, Kameshwar Poolla, and Tyrone L. Vincent. *Aggregate Flexibility of Thermostatically Controlled Loads. IEEE Transactions On Power Systems, Vol. 30, No. 1, Jan. 2015*

Conclusion

- **Part I: Mean field game is powerful tool for analyzing large games**
  - Mean field games $\Leftrightarrow$ Social Welfare Optimization Problem
    - **Efficiency evaluation**: Mean-field equilibrium coincides with social optimizer
    - **Existence, uniqueness, computation** of MFE through optimization

- **Part II: Geometric Approach for Aggregate Flexibility Modeling**
  - Individual flexibility can be represented by power consumption set (Polytope)
  - Aggregate flexibility modeling can be viewed as approximating Minkowski sum of heterogeneous polytopes
  - Include virtual battery modeling as special case

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- S. Lin, W. Zhang, J. Lian, and K. Kalsi, "Social Optima in Non-Cooperative Mean Field Games", CDC16
- S. Lin, W. Zhang, and L. Zhao, "Connections between mean-field games and social welfare optimization", under review
- L. Zhao, W. Zhang, H. Hao and K. Kalsi, "A Geometric Approach to Aggregate Flexibility Modeling of Thermostatically Controlled Loads", to appear in TPWRS

Thank you!